

Vivorio, Agostino

Sublimioris Geometriae Opuscula

Venetia 1772

4 Math.p. 377

urn:nbn:de:bvb:12-bsb10053920-2

Math. P.

377

4^o Math P 377

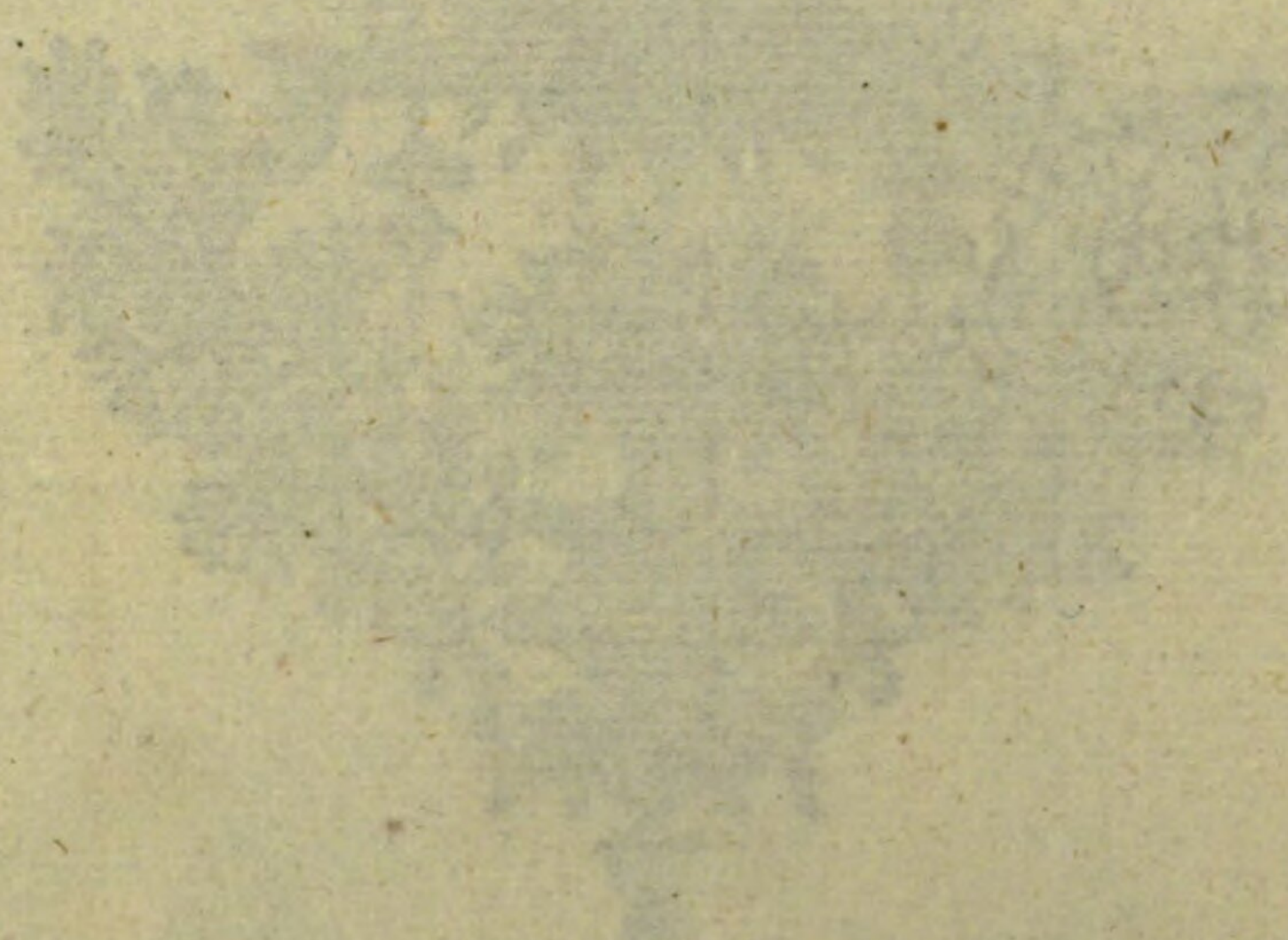
EX 155



SUBLIMIORIS GERMANIAE

ORDENI

AUGUSTINI



VENETIIS MDCCLXXII

REGIUM MAGNIFICENTIAE
MAGNIFICENTIAE

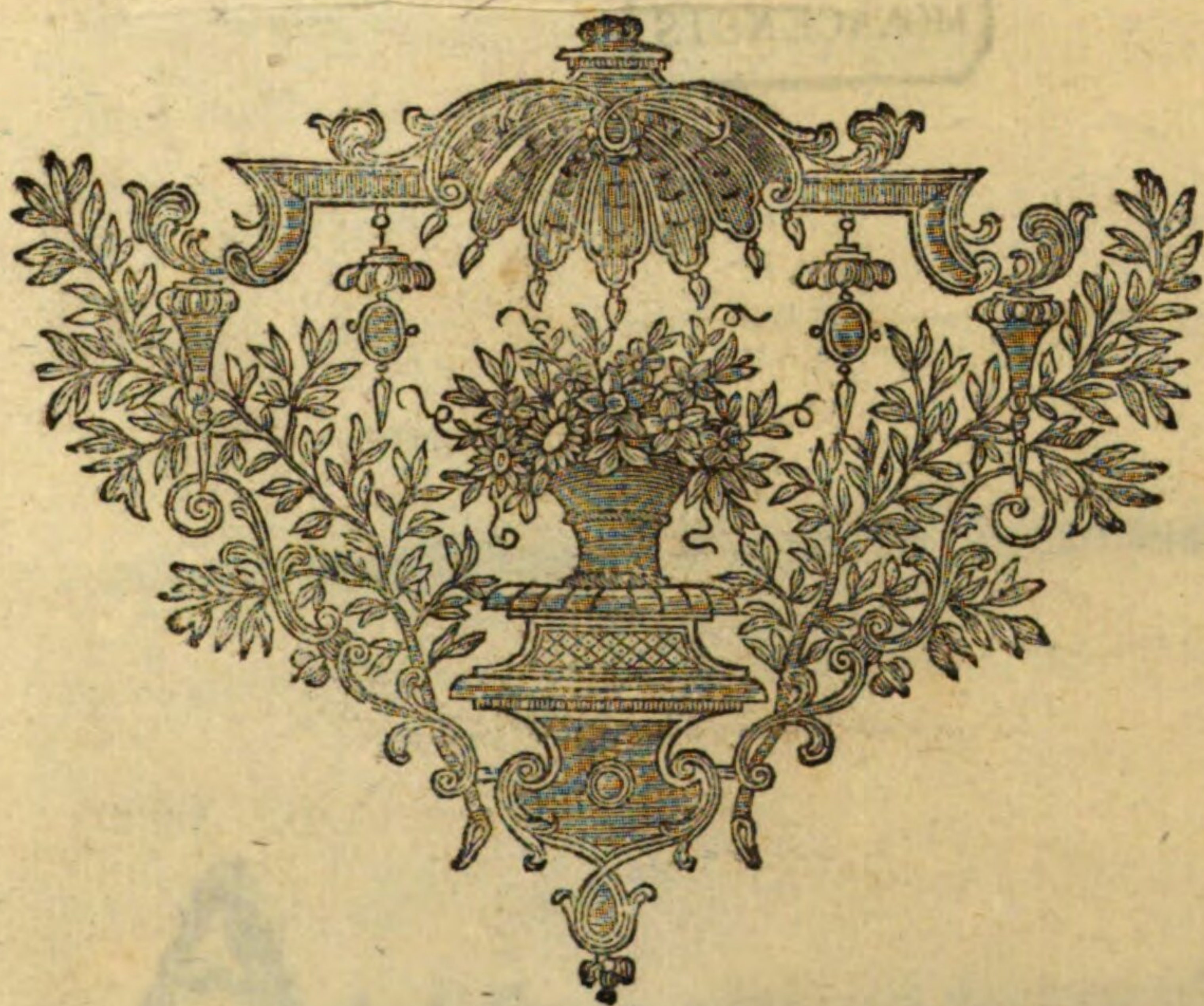
~~Nathaniel pag. 292.~~

R

SUBLIMIORIS GEOMETRIAE

OPUSCULA

AUGUSTINI VIVORII.



VENETIIS MDCCLXXII.

EXCUDEBAT MODESTUS FENTIUS
SUPERIORIBUS ANNUENTIBUS.

SUBLIMIORIS GEOMETRIAE

LIBER PRIMUS

DE LINEARIBUS

AUGUSTINI VIVORII

BIBLIOTHECA
REGIA
MONACENSIS.

VENERABILIS

LIBRARIUS

DE

LIBRARIIS

CELSISSIMO ADOLESCENTI
CO. GEORGIO POTOCKI

SARMATARUM REIPUBLICAE OPTIMATI,
DYMIRIENSI STAROSTAE &c. &c.

AUGUSTINUS VIVORIUS VICETINUS

S. P. D.

 *Cerbissimum animi mei dolorem afflictis Serenissimae Patriae Tuae rebus, CELSISIME ADOLESCENS, Tui adauget moeroris recordatio. Nam*

cum

cum naturae debeas animum ad misericordiam
proclivem , fieri nequaquam potest , ut tantis
bellorum fluctibus agitata Republica non ingemi-
scas ; praesertim cum de libertate , & Religio-
ne agatur , quarum in humana vita maxima
vis est . Sed dolorem quidem cuilibet in prom-
ptu est usitatis , & ex morali Philosophia de-
ductis argumentis temperare : saepius repetita fati
in rebus humanis potestate . Ego autem , qui
me Tibi plurimum debere profiteor , cum me ad-
junxeris in Comitum Tuorum numerum , qui-
buscum de arcanis Matbeseos in Italia colloque-
rere , aliam ineundam mihi esse rationem duxi ,
ut hunc animi Tui sensum lenirem , a Rei-
publicae ad studiorum curam egregiam mentem
Tuam avocando . Habeas itaque haec penitio-
ris Geometriae Opuscula , quae Tuo Nomini ,
Te annuente , inscribo , antiqui obsequii testes ,
&

& praesentibus accommodatum solatium tempori-
bus, quae Deus optimus maximus in melius
vertat. Vale.

D. Venetius Cal. Sept. A. 1772.

Amassons toujours des verités de Mathematique & de Physique au hazard de ce qui en arrivera , ce n' est pas risquer beaucoup . Il est certain qu' elles seront puisées dans un fonds d'où il en est déjà sorti un grand nombre qui se sont trouvées utiles . Nous pouvons présumer avec raison que de ce même fonds nous en tirerons plusieurs brillantes dès leur naissance, d'une utilité sensible, & incontestable . Il y en aura d' autres qui attendront quelque temps qu' une fine meditation, ou un heureux hazard découvre leur usage . Il y en aura , qui prises separément seront steriles , & ne cesseront de l' être , que quand on s' avisera de les rapprocher. M.^r de Fontenelle.

OPUSCULUM I.

MULTIPLEX PARABOLAE, ELLIPSIS, ET HYPERBOLAE DESCRIPTIO OPE SEMICIRCULI.



Circulus, proprietatibus, & figura Curvarum omnium elegantissimus, nedum independens a Cono est, quippe qui basis imo, & fundamentum sit ipsius Coni; sed omnes quoque Conicae Sectiones, quae ab hoc solido pendunt, per solum Circulum, exclusa Coni fabrica, nullo negotio eliciuntur, dummodo Circulus cum rectis lineis attentius conferatur. Ex hac enim mutua collatione, non unicam modo, sed varias describendarum Conicarum Sectionum rationes deduxi, quarum simpliciores, atque elegantiores, constructionibus geometricis illustratas, summa brevitate hoc in Opusculo exposui. Advertant autem sublimioris Geometriae Cultores, me in variis Conicarum Sectionum descriptionibus abscissarum initium in altera tantum diametri extremitate constanter collocare; cum quodlibet aliud punctum vel in ipsa diametro, vel intra semicirculum ad idem opus sumere licuisset. Ex quo facile colligent, tantum semicirculi ope cum rectis lineis collati, longe plures, quam ego indicaverim, emergere describendarum earum Sectionum rationes.

§. I.

Fig. 1. **D**iametro AB semicirculi ARB , cujus centrum C , fit normalis recta quaelibet DR , & jungatur BR , fiatque $BR : BC :: DB : DP$; locus, in quo sunt omnia puncta P eodem modo inventa, erit Parabola Apolloniana.

Sit radius $CB = a$, $BD = x$, & $DP = y$; erit $BR = \sqrt{2ax}$. Quare $\sqrt{2ax} : a :: x : y$; ideoque $y\sqrt{2ax} = ax$, & $2axy^2 = a^2x^2$, sive $\frac{1}{2}ax = y^2$, quae est aequatio ad Parabolam, cujus vertex punctum B , parameter vero $\frac{1}{2}a$. Et quoniam, posita abscissa $= 2a$, fit $y = a$; erit semiordinata AK aequalis radio semicirculi CB .

§. II.

Fig. 2. Si agatur RH tangens circulum in quolibet puncto R , atque accipiat recta DP media proportionalis inter CD , & BH ; erit locus punctorum P Parabola.

Etenim ducatur radius CR ; erit angulus CRH rectus, proindeque $CD : DR :: DR : DH$, nempe $a - x : \sqrt{(2ax - x^2)} :: \sqrt{(2ax - x^2)} : DH$; idcirco $BH = \frac{2ax - x^2}{a - x}$; $\frac{2ax - x^2}{a - x} = x$; Quare $y^2 = (a - x)x$

$\frac{ax}{a - x} = ax$; aequatio ad Parabolam, cujus parameter

3

meter aequatur radio CB. Quapropter si ex centro C erigatur perpendicularum CE, transibit Parabola per punctum E secans bifariam circumferentiam ARB.

§. III.

Si producat diameter AB in H, ita ut recta BH sit aequalis radio semicirculi, & agatur quaelibet recta HR occurrens semiperipheriae in R, fiatque DP aequalis ipsi HR; puncta P erunt in segmento Parabolae KE, cujus minima semiordinata erit aequalis radio, maxima vero aequabit triplum radium semicirculi genitoris.

Quoniam recta $DR = \sqrt{2ax - x^2}$, erit $HR = \sqrt{a^2 + 2ax + x^2 + 2ax - x^2} = \sqrt{4ax + a^2}$. Quapropter $DP = y = \sqrt{4ax + a^2}$, & $y^2 = 4ax + a^2$, quae aequatio ad Parabolam pertinet. Quia vero $4ax + a^2 = 4a(x + \frac{1}{4}a)$, erit parameter Parabolae $= 4a$, & abscissa integrae Parabolae $= x + \frac{1}{4}a$, cujus pars $\frac{1}{4}a$ erit constans. Quare si accipiatur $BQ = \frac{1}{4}a$, erit punctum Q apex Parabolae aequationis inventae. Praeterea quia recta HR in puncto B evadit HB, atque in alio A fit HA, erit minima semiordinata segmenti Parabolici BK aequalis radio, & maxima AE aequalis triplo radio semicirculi, seu AH. Et revera, si in aequatione ponatur $x = 0$, fit $y = a$; si $x = 2a$, evadit $y = 3a$.

§. IV.

Sit recta indefinita BK normalis extremitati diametri AB, & ex altera extremitate A ducatur quaelibet recta AE, quae occurrat ipsi BK; dein ex cen-

A 2

tro

Fig. 3.

Fig. 4.

tro C agatur CR parallela rectae AE, & ex puncto intersectionis H demittatur perpendicularum HD, fiatque $AE:EF::CH:DP$; puncta P tali pacto inventa erunt in Parabola, cujus vertex bifariam dividet dimidiam circumferentiam.

Quoniam $CD:CB::DH:BR$, erit $a - x :$

$$a :: \sqrt{(2ax - x^2)} : \frac{a\sqrt{(2ax - x^2)}}{a - x} = BR.$$

$$\text{Quare } BE = \frac{2a\sqrt{(2ax - x^2)}}{a - x}, \text{ \& } AE =$$

$$\sqrt{\left(4a^2 + \frac{8a^3x - 4a^2x^2}{(a - x)^2}\right)}. \text{ At vero } BE^2 = AE$$

$$\times EF; \text{ igitur } \frac{8a^3x - 4a^2x^2}{(a - x)^2} = AE \times EF =$$

$$EF \times \sqrt{\left(4a^2 + \frac{8a^3x - 4a^2x^2}{(a - x)^2}\right)}, \text{ \& } EF =$$

$$\frac{8a^3x - 4a^2x^2}{(a - x)^2 \times \sqrt{\left(4a^2 + \frac{8a^3x - 4a^2x^2}{(a - x)^2}\right)}}. \text{ Qua-}$$

$$\text{propter cum fit ex hypothefi } AE:EF::CH:$$

$$DP, \text{ erit } \sqrt{\left(4a^2 + \frac{8a^3x - 4a^2x^2}{(a - x)^2}\right)} :$$

$$\frac{8a^3x - 4a^2x^2}{(a - x)^2 \times \sqrt{\left(4a^2 + \frac{8a^3x - 4a^2x^2}{(a - x)^2}\right)}} :: a:y, \text{ \& }$$

$$(a - x)^2 \times \sqrt{\left(4a^2 + \frac{8a^3x - 4a^2x^2}{(a - x)^2}\right)} : 8a^3x$$

$$- 4a^2x^2 :: a:y; \text{ proindeque } 4a^2(2a^2x - ax^2)$$

$$= y(a - x)^2 \times \left(4a^2 + \frac{8a^3x - 4a^2x^2}{(a - x)^2}\right) =$$

$$4a^2y(a-x)^2 + \frac{4a^2y(a-x)^2 \times (2ax - x^2)}{(a-x)^2}$$

$$= 4a^2y(a-x)^2 + 4a^2y(2ax - x^2), \text{ \& } 2a^2x - ax^2 = a^2y - 2axy + yx^2 + 2axy - yx^2, \\ \text{feu } 2ax - x^2 = ay; \text{ aequatio ad Parabolam.}$$

Constructio autem aequationis facile invenietur, duobus adnotatis. Primum: tunc Parabolam in circumferentiam incurrere, cum recta AE est perpendicularis diametro AB; etenim hoc in casu CR erit quoque ipsi diametro normalis, adeoque, quia AE = EF, erit CH = DP, atque ipsamet DP incidet in rectam CH, consequenter Parabola occurret peripheriae in puncto Q, quod bifariam dividit dimidiam circumferentiam; id quoque ex ipsamet Analysis satis liquet, quia si ponatur $x = a$, emerget $y = a$. Alterum: punctum Q esse verticem Parabolae, quia in aequatione habetur quadratum x^2 . Hisce perspectis nullo negotio aequatio contrahitur. Etenim erigatur perpendiculum CQ, & ex puncto P agatur PG parallela radio BC. Et quoniam BD = x, DP = y, & parameter Parabolae = a, erit PG = a -

x , QG = a - y, & QG $\times$ a = GP², idest $a^2 - ay = a^2 - 2ax + x^2$, feu $ay = 2ax - x^2$. Caeterum animadvertendum est, ab eadem aequatione nedum semiparabolam QB, verum etiam alteram oppositam QA, eodem stante abscissarum initio, definiri. Nam si fiat $x = 0 = \frac{1}{2}a = \frac{2}{3}a$, proveniet $y = 0 = \frac{3}{4}a = \frac{8}{9}a$; similiterque fit $y = 0 = \frac{3}{4}a = \frac{8}{9}a$, si ponatur $x = 2a = \frac{3}{2}a = \frac{4}{3}a$. Et revera si describatur semicirculus AIN alteri AQB aequalis, recta AL angulum obtusum cum diametro AB comprehendens dimidiam circumferentiam AIN fecabit, atque perpendiculo NL occurret, ejusque parallela erit CS, & OT quarta proportionalis post AL, LI, & CS,

& CS, extremitatesque T rectarum OT erunt in semiparabola QA. Erit enim $TV = OC = BO = BC = x - a$; ideoque $x^2 - 2ax + a^2 = a^2 - ay$, nempe $2ax - x^2 = ay$: hujus rei ratio satis liquet, cum quadratum $a^2 - 2ax + x^2$ emergat tam ex radice $a - x$, quam ex $x - a$. Quare si angulus BAE erit acutus, habebitur descriptio semiparabolae QB; si rectus ut BAM, vertex Parabolae Q; denique si obtusus ut BAL, opposita semiparabola QA emerget: atque ita integro radii CB circuitu per totam semiperipheriam BQA proveniet Parabola BQA, cujus parameter ipsemet radius CB.

§. V.

Fig. 5. Si erigantur quaelibet perpendiculares DR, & jungantur BR, accipianturque rectae Dp, quae sint aequales rectis BR, seu in qualibet constanti ratione cum iisdem BR; erit locus punctorum p Parabola.

Etenim sit $\frac{m}{n}$ aequalis, vel major, seu minor unitate. Erit itaque $Dp = \frac{m}{n} \sqrt{2ax}$, consequenter $y = \frac{m}{n} \sqrt{2ax}$, & $y^2 = \frac{2m^2}{n^2} ax$, quae est ad Parabolam Be, cujus parameter $\frac{2m^2}{n^2} a$, vertex punctum B, & maxima semiordinata Ae $= \frac{2m}{n} a$.

Quod si in iisdem perpendiculis DR sumantur rectae DP, quae habeant eandem constantem rationem cum oppositis chordis AR, quam habent Dp cum aliis

7

aliis BR ; puncta P erunt in Parabola AK , quae erit alteri Be aequalis, cujus initium abscissarum erit opposita diametri extremitas A . Quapropter quia applicatae Dp ad chordas BR relationem habent, sicut pariter DP ad AR , vertex Parabolae illarum erit punctum B , harum vero A .

Sit $\frac{m}{n} = \frac{4}{3}$; igitur $y^2 = \frac{3^2}{9} ax$. Fiat $x = a$; erit $y^2 = \frac{3^2}{9} a^2$, & $y = \frac{4}{3} a \sqrt{2}$; quare semiordinata $CQ = \frac{4}{3} a \sqrt{2}$. Sit $x = 2a$; emerget $y^2 = \frac{6^2}{9} a^2$, & $y = \frac{8}{3} a = AE$.

Sit $\frac{m}{n} = 1$; erit $y^2 = 2ax$. Ponatur $x = a = 2a$; exurget $y^2 = 2a^2 = 4a^2$, adeoque $y = a\sqrt{2} = 2a$, consequenter $CQ = a\sqrt{2}$, & $AE = 2a$.

Denique sit $\frac{m}{n} = \frac{1}{2}$; proveniet $y^2 = \frac{1}{2}ax$. Fiat $x = a = 2a$; exurget $y^2 = \frac{1}{2}a^2 = a^2$, & $y = a\sqrt{\frac{1}{2}} = a$. Igitur $Cq = a\sqrt{\frac{1}{2}}$, & $Ae = a$.

Ex datis peculiaribus circumstantiis, quibus adnexa est quaelibet haecenus descripta Parabola, enascuntur peculiares item, & variae hujus Curvae proprietates, quas perspicax, & impiger Geometra pro otio vel voluptate elicere per meditationem posset. Cujus rei specimen exhibere satis erit in nonnullis symptomatis, quae in Parabolae descriptione ultimo loco exhibitae, cujus parameter aequatur dimidio radio semicirculi genitoris, advertere libuit; idque in exemplum caeterarum. Non enim tempus est omnia considerare, & exponere. Quod tum de Parabola, tum de reliquis, quae consequuntur, Curvis dictum volo.

I. Si ex puncto intersectionis Parabolae, atque circumferentiae F demittatur perpendicularum FG , hoc bi-

bifariam dividet radium AC . Etenim $\overline{FG} = BG \times AG$, & idem $\overline{FG} = \frac{1}{2}a \times BG = \frac{1}{2}AC \times BG$; quare $BG \times AG = \frac{1}{2}AC \times BG$, & $AG = \frac{1}{2}AC$.

II. Recta CH , quae jungit centrum C , & punctum H intersectionis applicatae GF , atque Parabolae AK , aequatur semiordinatae Cq . Est enim $Cq = AC \times CG$, & $\overline{CH} = \overline{CG} + \overline{GH} = 2\overline{CG} = 2CG \times CG = AC \times CG$; unde $\overline{Cq} = \overline{CH}$, & $Cq = CH$.

III. Si agatur FL tangens circulum in puncto intersectionis circumferentiae, & Parabolae F , erigaturque ex puncto L perpendicularum LM , cujus quadratum sit dimidium quadrati tangentis FL ; punctum M erit in Parabola Be producta. Nam $GC : CF ::$

$CF : CL$, ideoque $CA = AL$; quare $\overline{FL} = \overline{FG} + \overline{GL} = AG \times GB + \overline{GB} = GB \times (AG + GB) = AB \times BG = 4AG \times GB = 2AG \times 2GB = 2AG \times BL$. At vero ex hypothese $LM = AG \times BL$; igitur punctum M est in Parabola BM .

IV. Jungatur AF , & sit GS normalis rectae AF , itemque ST diametro; punctum T erit focus Parabolae AK . Etenim $AF = a$, $FG = \sqrt{\frac{3}{4}a^2}$, & $AF : FG :: FG : FS$, idest $a : \sqrt{\frac{3}{4}a^2} :: \sqrt{\frac{3}{4}a^2} : \frac{3}{4}a = FS$; ideoque $AS = AF - SF = a - \frac{3}{4}a = \frac{1}{4}a$; quare $AG : AS :: AS : AT$, nempe $\frac{1}{2}a : \frac{1}{4}a :: \frac{1}{4}a : \frac{1}{8}a = AT$, consequenter punctum T est focus Parabolae AK .

ELLIPSIS DESCRIPTIO.

§. I.

SI recta DR diametro AB semicirculi ARB perpendicularis secetur in P secundum quamlibet rationem datam; puncta P eodem modo determinata erunt in Ellipsi Apolloniana. Fig. 6.

Nam fit $BD = x$, $DP = y$, radius $BC = a$, atque ratio data m ad n . Itaque erit $DR = \sqrt{(2ax - x^2)}$, & $PR = \sqrt{(2ax - x^2)} - y$. Igitur $DP : PR :: m : n$, idest $y : \sqrt{(2ax - x^2)} - y :: m : n$; hinc $ny = m\sqrt{(2ax - x^2)} - my$, $ny + my = m\sqrt{(2ax - x^2)}$, $n^2y^2 + 2nmy^2 + m^2y^2 = 2m^2ax - m^2x^2$, denique $(n^2 + 2nm + m^2)y^2 = (2m^2a - m^2x)x$, quae est aequatio ad Ellipsin. Ponatur $x = 0$; erit $(n^2 + 2nm + m^2)y^2 = 0$, & $y = 0$: itidem fiat $x = 2a$; emergit $(n^2 + 2nm + m^2)y^2 = 4m^2a^2 - 4m^2a^2 = 0$, & $y = 0$. Quare axis major Ellipsis erit $2a$. Fiat modo $x = a$; erit $(n^2 + 2nm + m^2)y^2 = 2m^2a^2 - m^2a^2 = m^2a^2$,

$$(n + m)y = ma, \text{ \& } y = \frac{ma}{n + m}; \text{ ideoque } \frac{2ma}{n + m} =$$

$$\text{axi minori. Tandem } 2a : \frac{2ma}{n + m} :: \frac{2ma}{n + m} :$$

$\frac{2m^2a}{(n + m)^2} =$ parametro Ellipsis. Et revera in Ellipsi quadratum semiordinatae est ad rectangulum ex segmentis axis, ut parameter ad axem, & conse-

$$\text{quenter } y^2 : 2ax - x^2 :: \frac{2m^2a}{(n + m)^2} : 2a; \text{ hinc}$$

$$2ay^2 = \frac{2m^2a}{(n + m)^2} \times (2ax - x^2), \quad 2m^2a(2ax -$$

$\frac{— x^2}{y^2(n+m)^2} = 2 a y^2 (n+m)^2, m^2(2 a x — x^2) =$
 $y^2(n+m)^2, \& (2 m^2 a — m^2 x) x = (n^2 + 2 n m +$
 $m^2) y^2, \text{ prorsus ut supra.}$

Quoniam vero aequatio in hanc resolvitur analogiam $n^2 + 2 n m + m^2 : m(2 a — x) :: m x : y^2$, erit quadratum summae binorum terminorum datae rationis ad rectangulum ex axe maiore abscissa multiplicato in antecedentem datae rationis, ut rectangulum ex abscissa in praefatum antecedentem ad quadratum semiordinatae.

Sit $m : n :: 1 : 1$; erit $4 y^2 = 2 a x — x^2$. Quare axis minor erit a , & parameter $\frac{1}{2} a$. Itaque Ellipsis secat bifariam radium CK in puncto N , & maxima semiordinata CN aequatur parametro.

Sit $m : n :: 2 : 1$; emerget $9 y^2 = 8 a x — 4 x^2$. Igitur erit axis minor $\frac{4}{3} a$, parameter vero $\frac{8}{9} a$.

Quod si ponatur $m : n :: 1 : 0$, Ellipsis degenerabit in Circulum; habebitur enim $y^2 = 2 a x — x^2$, quae ad Circulum pertinet. Et si fiat $m : n :: 0 : 1$, locus punctorum P erit diameter BA , quia $y = 0$.

§. II.

Si normalis RD ultra diametrum continuetur in Q , ita ut sit $RD : DQ :: m : n$; puncta Q tali pacto inventa erunt in Ellipsi.

Etenim erit $\sqrt{(2 a x — x^2)} : y :: m : n$; proindeque $m y = n \sqrt{(2 a x — x^2)}$, & $m^2 y^2 = 2 n^2 a x — n^2 x^2$, quae ad Ellipsin spectat. Ponatur $x = 0 = 2 a$; emerget $y = 0 = 0$, consequenter $2 a$, seu BA erit axis major, si $m > n$; minor vero, si $m < n$. Fiat modo $x = a$; erit $m^2 y^2 = 2 n^2 a^2 — n^2 a^2 = n^2 a^2$, $y^2 = \frac{n^2}{m^2} a^2$, & $y = \frac{n}{m} a$; ideoque $\frac{2 n}{m} a$ erit axis
mi.

minor, five major, prout m major, vel minor fuerit quantitate n . Denique $2a : \frac{2n}{m}a :: \frac{2n}{m}a :$

$\frac{2n^2}{m^2}a$; quare $\frac{2n^2}{m^2}a$ erit parameter Ellipsis.

Sit $m > n$. Fiat $y = \frac{n^2}{m^2}a$; exurget $\frac{m^2 n^4}{m^4}a^2 = 2n^2ax - n^2x^2$, $m^2x^2 - 2m^2ax = -n^2a^2$,
 $x^2 - 2ax + a^2 = \frac{m^2a^2 - n^2a^2}{m^2}$, & $x = a \pm$

$\frac{a}{m} \sqrt{(m^2 - n^2)}$, proindeque BF distantia alterius

foci a vertice $= a - \frac{a}{m} \sqrt{(m^2 - n^2)}$, alte-

rius autem Bf $= a + \frac{a}{m} \sqrt{(m^2 - n^2)}$.

Sit $m : n :: 5 : 3$; erit $25y^2 = 18ax - 9x^2$, consequenter axis minor $= \frac{6}{5}a$, & parameter $= \frac{18}{25}a$; distantia vero foci a vertice BF $= \frac{1}{5}a$, & Bf $= \frac{9}{5}a$.

Quod si fuerit $m < n$, tum nullo negotio invenietur parameter axis majoris, & ideo distantia foci a vertice ope communis formulae.

§. III.

Si perpendicularum DR producat in H, ita ut sit DR : RH :: $m : n$; puncta H eodem modo determinata erunt in Ellipsi, cujus axis minor erit diameter BA semicirculi BRA.

Erit enim RH $= y - \sqrt{(2ax - x^2)}$; ideoque $\sqrt{(2ax - x^2)} : y - \sqrt{(2ax - x^2)} :: m : n$,
 B 2 &

$\& n \sqrt{(2ax - x^2)} = my - m \sqrt{(2ax - x^2)},$
 $(n + m) \sqrt{(2ax - x^2)} = my, m^2 y^2 = (m^2 + 2mn + n^2) 2ax - (m^2 + 2mn + n^2) x^2;$ aequatio ad
 Ellipsin. Fiat $x = a$; erit $m^2 y^2 = (m^2 + 2mn + n^2) 2a^2 - (m^2 + 2mn + n^2) a^2 = (m^2 + 2mn + n^2) a^2,$
 $\& my = (m + n) a;$ adeoque $y = \frac{(m + n)}{m} a,$
 $\&$ axis major Ellipsis $\frac{2(m + n)}{m} a$; minor vero
 $2a$. Denique $2a : \frac{2(m + n)}{m} a :: \frac{2(m + n)}{m} a :$
 $\frac{2(m + n)^2}{m^2} a =$ parametro axis conjugati.

Sit $m : n :: 2 : 1$; erit $4y^2 = 18ax - 9x^2$.
 Igitur axis major $3a$, minor $2a$, & parameter $\frac{9}{2}a$.

Quoniam tres superiores descriptiones ope rectarum DR, QR, & DH secundum datam rationem sectarum in P, D, & R inveniuntur, hinc quandam inter se habent relationem, etsi pluribus aliis rationibus discrepent.

§. IV.

Fig. 7. Si producat diameter AB, & sumatur quaelibet BC minor BA, describaturque semicirculus AEC; dein erigatur perpendicularum BE, & protrahatur in F, ita ut sit BF aequalis diametro BA; tum ex quolibet puncto circumferentiae R demittatur normalis RD, & jungantur RB, RF, sintque EG, GP parallelae rectis FR, BD: puncta P eodem pacto inventa erunt in Ellipsi, cujus axis major erit BA, minor BE, & parameter BC.

Sit $BE = b$; erit $BF : BE :: BR : BG$, nempe
 $2a :$

$2a : b :: \sqrt{2ax} : \frac{b\sqrt{2ax}}{2a} = BG$. At vero etiam

$DR : DP :: BR : BG$, idest $\sqrt{(2ax - x^2)} : y ::$
 $\sqrt{2ax} : \frac{b\sqrt{2ax}}{2a}$.

Igitur $y\sqrt{2ax} = \frac{b\sqrt{2ax} \times \sqrt{(2ax - x^2)}}{2a}$, $2ay\sqrt{2ax} = b\sqrt{2ax} \times$

$\sqrt{(2ax - x^2)}$, $2ay = b\sqrt{(2ax - x^2)}$, & $4a^2y^2 = 2ab^2x - b^2x^2$, quae est aequatio ad Ellip-
 psin. Fiat $x = 0 = 2a$; emergit $y = 0 = 0$, adeo-
 que $2a$ erit axis major Ellipsis. Sit $x = a$; erit
 $4a^2y^2 = 2a^2b^2 - a^2b^2 = a^2b^2$; proindeque $4y^2 = b^2$, & $y = \frac{1}{2}b$; quare $b = BE$ erit axis minor.

Et quoniam $2a : b :: b : \frac{b^2}{2a} = BC$, erit BC pa-
 rameter Ellipsis.

Superior aequatio hac quoque methodo eruitur.
 Quoniam $AB : BE :: BE : BC$; erit $AB : BC ::$

$\overline{AB} : \overline{BE} :: \overline{BF} : \overline{BE} :: \overline{BR} : \overline{BG} :: \overline{DR} : \overline{DP} :: AD$

$\times DB : DP$; hinc $AB : BC :: AD \times DB : DP$. At

vero $BC = \frac{b^2}{2a}$; ideoque $2a : \frac{b^2}{2a} :: 2ax - x^2 :$

y^2 , consequenter $2ay^2 = \frac{b^2}{2a} (2ax - x^2)$, &

$4a^2y^2 = 2ab^2x - b^2x^2$.

§. V.

Quod si sumatur BC major diametro BA , & BF Fig. 8.
 ipsi BA aequalis; tum ex puncto F agatur recta
 FR , atque ex alio puncto E recta EG priori FR
 paral-

parallela, quae occurrat chordae BR protractae in G, sitque GP parallela diametro BA in DR productam incurrens: puncta P dabunt locum Ellipsis, cujus axis major erit BE, minor BA, parameter vero BC.

Nam hic quoque $BF:BE::BR:BG$, seu $2a:b::\sqrt{2ax}:\frac{b\sqrt{2ax}}{2a}=BG$. Deinde, quia tri-

goni RDB, RPG sunt similia, erit $RG:RB::PR:RD$, & componendo $GB:RB::PD:RD$.

Igitur erit $\frac{b\sqrt{2ax}}{2a}:\sqrt{2ax}::y:\sqrt{2ax-x^2}$;

consequenter $y\sqrt{2ax}=\frac{b\sqrt{2ax}}{2a}\times\sqrt{2ax-x^2}$,

$y=\frac{b}{2a}\sqrt{2ax-x^2}$, $2ay=b\sqrt{2ax-x^2}$,

& $4a^2y^2=2ab^2x-b^2x^2$ ad Ellipsin pertinens.

Quamobrem $b=BE$ erit axis major Ellipsis, & BA minor; nam cum sit BE major BF, erit quoque major BA, cui ipsamet BF aequatur. Quoniam vero $AB:BE::BE:BC$, erit BC parameter axis conjugati.

Si vero accipiatur BC aequalis diametro BA, tunc locus punctorum P erit eadem semiperipheria ARB, quandoquidem recta BF erit aequalis rectae BE, proindeque bina puncta G, & P incident in R.

C A P U T III.

HYPERBOLAE DESCRIPTIO.

§. I.

Producat diameter AB in N, ita ut sit BN *Fig. 9.*
 aequalis radio semicirculi, atque fiat ut dupla
 diameter, nempe 2 AB ad ND, ita quadratum rectae
 NK ad quadratum normalis DP; puncta P erunt
 in segmento Hyperbolae Apollonianaes, & aequilaterae
 RE.

Nam sit de more $AB = 2a$, $BD = x$, $DP = y$.

Erit itaque $\overline{NK} = \overline{ND} + \overline{DK} = a^2 + 2ax + x^2 + 2ax - x^2 = a^2 + 4ax$. Quare $4a : a + x :: a^2 + 4ax : y^2$, & consequenter $4ay^2 = a^3 + a^2x + 4a^2x + 4ax^2 = a^3 + 5a^2x + 4ax^2$, $4y^2 = a^2 + 5ax + 4x^2$, & $y^2 = x^2 + \frac{5}{4}ax + \frac{1}{4}a^2$, quae ad Hyperbolam aequilateram pertinet. Praeterea cum sit $x^2 + \frac{5}{4}ax + \frac{1}{4}a^2 = (x + \frac{1}{4}a) \times (x + a) = (x + \frac{1}{4}a) \times (x + \frac{1}{4}a + \frac{3}{4}a)$, distantia inter verticem Hyperbolae, & initium abscissarum erit $\frac{1}{4}a$, & axis, sive parameter $\frac{3}{4}a = HN$, quandoquidem in Hyperbola aequilatera quaelibet semiordinata est media proportionalis inter distantiam verticis Hyperbolae ab ipsamet semiordinata, atque rectam compositam ex eadem distantia, & axe. Fiat $x = 0 = a = 2a$; evadit $y = \frac{1}{2}a = \frac{1}{2}a\sqrt{10} = \frac{1}{2}a\sqrt{27}$; igitur $BR = \frac{1}{2}a$, $CQ = \frac{1}{2}a\sqrt{10}$, & $AE = \frac{1}{2}a\sqrt{27}$. Caeterum Hyperbolam nequaquam semiperipheriae occurrere satis liquet; enim vero si ponatur $2ax - x^2 = x^2 + \frac{5}{4}ax + \frac{1}{4}a^2$, emergit $x = \frac{3}{16}a \pm \sqrt{-\frac{23}{256}a^2}$.

§. II.

§. II.

Fig. 10. Si fumatur in perpendicularo DK recta DP, cujus quadratum eandem habeat relationem ad quadratum chordae BK, quam habet recta composita ex radio BC, & abscissa BD ad triplum radium; puncta P erunt in Hyperbola scalena, cujus vertex erit extremitas diametri B, & maxima semiordinata AE aequalis diametro BA.

Etenim erit $y^2 : 2ax :: a+x : 3a$; proindeque $3ay^2 = 2a^2x + 2ax^2$, & $y^2 = \frac{2}{3}ax + \frac{2}{3}x^2$ ad Hyperbolam scalenam pertinens. At vero $ay^2 = \frac{2}{3}a^2x + \frac{2}{3}ax^2$; ideoque $y^2 : (a+x)x :: \frac{2}{3}a : a$. Quapropter, quia in Hyperbola scalena quadratum semiordinatae est ad rectangulum ex abscissa in rectam compositam ex eadem abscissa, & axe transverso, ut parameter ad eundem axem, erit a , seu radius axis transversus Hyperbolae, & $\frac{2}{3}a$ ejus parameter. Praeterea cum sit $a : a\sqrt{\frac{2}{3}} :: a\sqrt{\frac{2}{3}} : \frac{2}{3}a$, erit axis conjugatus Hyperbolae $= a\sqrt{\frac{2}{3}}$. Ponatur $x = 0 = a =$

$2a$; evadit $y = 0 = \frac{2a}{\sqrt{3}} = 2a$; consequenter punctum B est apex Hyperbolae, & semiordinata AE $= AB$, alia vero $CQ = \frac{2a}{\sqrt{3}}$. Ut autem occur-

fus Hyperbolae cum semiperipheria innotescant, fiat $\frac{2}{3}ax + \frac{2}{3}x^2 = 2ax - x^2$; evadit $x = \frac{2}{5}a \pm \frac{2}{5}a = \frac{4}{5}a = 0$. Quare Hyperbola occurrit peripheriae in puncto B, in quo $x = 0$, atque insuper in N, cujus abscissa BH $= \frac{4}{5}a$. Denique sit $y = \frac{1}{3}a$; emerget $\frac{1}{9}a^2 = \frac{2}{3}ax + \frac{2}{3}x^2$, $x^2 + ax + \frac{1}{4}a^2 = \frac{5}{12}a^2$, & $x = -\frac{1}{2}a \pm \sqrt{\frac{5}{12}a^2}$, quae erit distantia foci a vertice FB.

§. III.

§. III.

Agatur quaelibet NK tangens semicirculum BKA, *Fig. II.*
& ex puncto K demittatur perpendicularum KD, sit-
que DP tertia continue proportionalis ad CN, & BN;
puncta P eodem modo determinata erunt in Hy-
perbola BP.

Quoniam DN est tertia proportionalis ad CD, &
DK, erit eadem $DN = \frac{2ax - x^2}{a - x}$; consequenter

$$CN = \frac{a^2}{a - x}, \text{ \& } BN = \frac{ax}{a - x}. \text{ Quare } CN : BN ::$$

$$BN :: DP, \text{ nempe } \frac{a^2}{a - x} : \frac{ax}{a - x} :: \frac{ax}{a - x} : y;$$

$$\text{ideoque } \frac{a^2 y}{a - x} = \frac{a^2 x^2}{a^2 - 2ax + x^2}, \text{ \& } a^2 y = \frac{a^2 x^2}{a - x},$$

& $a^3 y - a^2 xy = a^2 x^2$, seu $ay - xy = x^2$, quae
est aequatio ad Hyperbolam inter affymptota. Fiat

$$x = a; \text{ erit } ay - ay = a^2, \text{ idest } y = \frac{a^2}{a - a} =$$

$$\frac{a}{1 - 1} = \frac{a}{0} = \infty; \text{ quare semiordinata CQ est}$$

infinita. Quamobrem producat eadem QC in R,
ita ut sit CR aequalis diametro BA, rectaque CR
dividatur bifariam in H, atque jungatur HB, cui
sit parallela RE indefinita: rectae RQ, RE erunt
affymptota Hyperbolae BP. Etenim ex puncto B
agatur BM parallela rectae CR; itaque $RM = HB$
 $= \sqrt{2}a^2$, & $MB = a$, consequenter $RM \times MB =$
 $a\sqrt{2}a^2 = a^2\sqrt{2}$; deinde $BC : DC :: BH : GH$,

$$\text{nempe } a : a - x :: a\sqrt{2} : \frac{(a - x)a\sqrt{2}}{a} = (a - x)$$

C

 $\times \sqrt{2}$

$x\sqrt{2} = GH = FR$, & $FP = FG + GD + DP =$
 $a + x + y$, quandoquidem $BC : CH :: BD : DG$,
 adeoque $BD = DG$. Quare si ponatur $RM \times MB$
 $= RF \times FP$; emerget $a^2\sqrt{2} = (a - x)\sqrt{2} \times (a +$
 $x + y)$, $a^2 = (a - x)(a + x + y) = a^2 + ax + ay$
 $- ax - x^2 - xy$, & $ay - xy = x^2$, prorsus ut
 supra. Quapropter rectangulum constans Hyperbolae
 BP erit $a^2\sqrt{2}$; proindeque si accipiat recta RO,
 quae sit media proportionalis inter RM, & MB, &
 agatur OL parallela asymptoto RQ; erit punctum
 L apex Hyperbolae, & LO = OR latus potentiae.
 Denique advertendum angulum asymptotorum R ef-
 se dimidium recti, quia aequatur angulo externo
 CHB.

§. IV.

Fig. 12. Sit BL aequalis radio BC, & ex puncto L aga-
 tur quaelibet recta LK, cui sit normalis KH, at-
 que ex puncto K demittatur perpendicularum KD, su-
 maturque DP aequalis interceptae DH; puncta P
 erunt in Hyperbola BPA.

Erit enim $LD : DK :: DK : DH$; ideoque $a + x :$
 $\sqrt{2ax - x^2} :: \sqrt{2ax - x^2} : \frac{2ax - x^2}{a + x} =$
 DH. At vero DH aequatur applicatae DP. Igitur
 $y = \frac{2ax - x^2}{a + x}$, & $ay + xy = 2ax - x^2$, quae est
 ad Hyperbolam inter asymptota.

Hanc aequationem Leonardo Salimbenio in Mili-
 tari Collegio Veronensi Alumnorum omnium lectissi-
 mo, dum Mathematicis institutionibus, docente cele-
 berrimo Professore Antonio-Mario Lorgna, Viro mi-
 hi aeternum diligendo, & celebrando, simul imbue-
 bamur,

bamur, construendam exhibui, qui sequentem elegantissimam constructionem solerter invenit, quam illi me fateor debere. Ex puncto L erigatur perpendiculum LR aequale duplae diametro BA, & producat BA in T, ita ut sit AT aequalis radio CA, jungaturque RT; erunt RL, RT asymptota Hyperbolae. Enim vero ex B ducatur recta BE parallela rectae TR, similiterque PF, & sit PG normalis rectae LR. Quoniam vero LT aequatur LR, etiam LB erit aequalis rectae LE; ideoque $LE = a$, $ER = 3a$, & $BE = \sqrt{2a^2}$. Deinde trigona PGF, TLR sunt similia, consequenter $PG = GF = DL = a + x$, & $PF = \sqrt{(2a^2 + 4ax + 2x^2)}$. Quapropter $BE \times ER = PF \times FR$, seu $3a\sqrt{2a^2} = \sqrt{(2a^2 + 4ax + 2x^2)} \times FR$; ideo

$$FR = \frac{3a\sqrt{2a^2}}{\sqrt{(2a^2 + 4ax + 2x^2)}} = 3a\sqrt{\left(\frac{2a^2}{2a^2 + 4ax + 2x^2}\right)} = 3a\sqrt{\left(\frac{a^2}{a^2 + 2ax + x^2}\right)}$$

$$= \frac{3a^2}{a + x}. \text{ At vero } RF + FG = RG = \frac{3a^2}{a + x} + a + x$$

$$= \frac{3a^2 + a^2 + 2ax + x^2}{a + x} = \frac{4a^2 + 2ax + x^2}{a + x}, \text{ \& } LG$$

$$= DP = y. \text{ Igitur } y = 4a - \frac{4a^2 + 2ax + x^2}{a + x}, \text{ \& }$$

$$ay + xy = 4a^2 + 4ax - 4a^2 - 2ax - x^2, \text{ seu } ay + xy = 2ax - x^2. \text{ Ita Condiscipulus implicatam aequationem extricavit.}$$

Ponatur $x = 0 = a = 2a$; exurget $y = 0 = \frac{1}{2}a = 0$. Quare Hyperbola occurrit semiperipheriae in punctis B, & A; ordinata vero CN, quae ex centro ducitur, aequatur dimidio radio. Ut autem maxima applicata OM inveniatur, differentiale aequationis reperiatur, quod erit $a dy + x dy + y dx = 2a dx$

$-2x dx$, in qua si ponatur $dy = 0$, proveniet $y dx$
 $= 2a dx - 2x dx$, & $y = 2a - 2x$, seu $\frac{2ax - x^2}{a+x}$
 $= 2a - 2x$, $2ax - x^2 = 2a^2 - 2ax + 2ax -$
 $2x^2$, $x^2 + 2ax = 2a^2$, & $x = a\sqrt{3} - a$. Porro
 $y = 2a - 2x$, ideoque maxima applicata $y = OM$
 $= 2a - 2a\sqrt{3} + 2a = 4a - 2a\sqrt{3}$, cujus ab-
 scissa $BO = a\sqrt{3} - a$. Praeterea sedulo perpenden-
 dum, verticem Hyperbolae inter puncta B, & M exi-
 ltere: agatur enim MQ parallela rectae TR, & MS
 normalis asymptoto LR; erit itaque $MQ = 2MS$
 $= 2x(a + a\sqrt{3} - a)^2 = 6a^2$, & $MQ = a\sqrt{6}$.
 At vero constans rectangulum $= 3a^2\sqrt{2}$, consequen-
 ter latus potentiae Hyperbolae $= \sqrt{3a^2\sqrt{2}} < a\sqrt{6}$,
 idest minor recta MQ. Tandem quia $LR = LT$,
 & angulus RLT rectus est, angulus asymptotorum
 semirectus erit.

§. V.

Fig. 13. Positis quae in antecedente §., si sumatur DP ae-
 qualis hypotenuse LH; puncta P erunt in Hyper-
 bola aequilatera MPQ.

Nam erit $LK = DL + DK = a^2 + 2ax + x^2 +$
 $2ax - x^2 = a^2 + 4ax$; proindeque $LK = \sqrt{a^2 + 4ax}$.
 At vero $DL : LK :: LK : LH$, nempe
 $a+x : \sqrt{a^2 + 4ax} :: \sqrt{a^2 + 4ax} : \frac{a^2 + 4ax}{a+x} =$
 LH . Quamobrem $y = \frac{a^2 + 4ax}{a+x}$, idest $ay + xy =$
 $a^2 + 4ax$, quae est ad Hyperbolam inter asym-
 ptota.

Ex

Ex puncto L erigatur perpendicularis LR aequalis duplae diametro BA, cui sit normalis indefinita RF; dein accipiatur BE aequalis quartae parti radii BC: dico rectas RL, RF esse asymptota Hyperbolae MPQ, & eandem Hyperbolam transire per punctum E. Enimvero agatur PG parallela rectae DL; erit ideo $PG = DL = a + x$, & $RG = RL - GL = RL - PD = 4a - y$. At vero quia BE aequatur quartae parti radii, idest $\frac{1}{4}a$, erit $EL = \frac{3}{4}a$. Quapropter $RG \times GP = RL \times LE$, seu $(4a - y)(a + x) = 4a \times \frac{3}{4}a$, $4a^2 - ay + 4ax - xy = \frac{3}{2}a^2$, $4ax + 4a^2 - 3a^2 = ay + xy$, & $4ax + a^2 = ay + xy$, ut supra. Fiat $x = 0 = a = 2a$; emergit $y = a = \frac{5}{2}a = 3a$, ideoque minima ordinata BM aequatur radio BC, & maxima AQ erit aequalis rectae AL. Tandem si dividatur angulus LRF bifariam recta RV; erit V apex Hyperbolae, & $VS = SR = a\sqrt{3}$ latus potentiae.



Fig. 1.

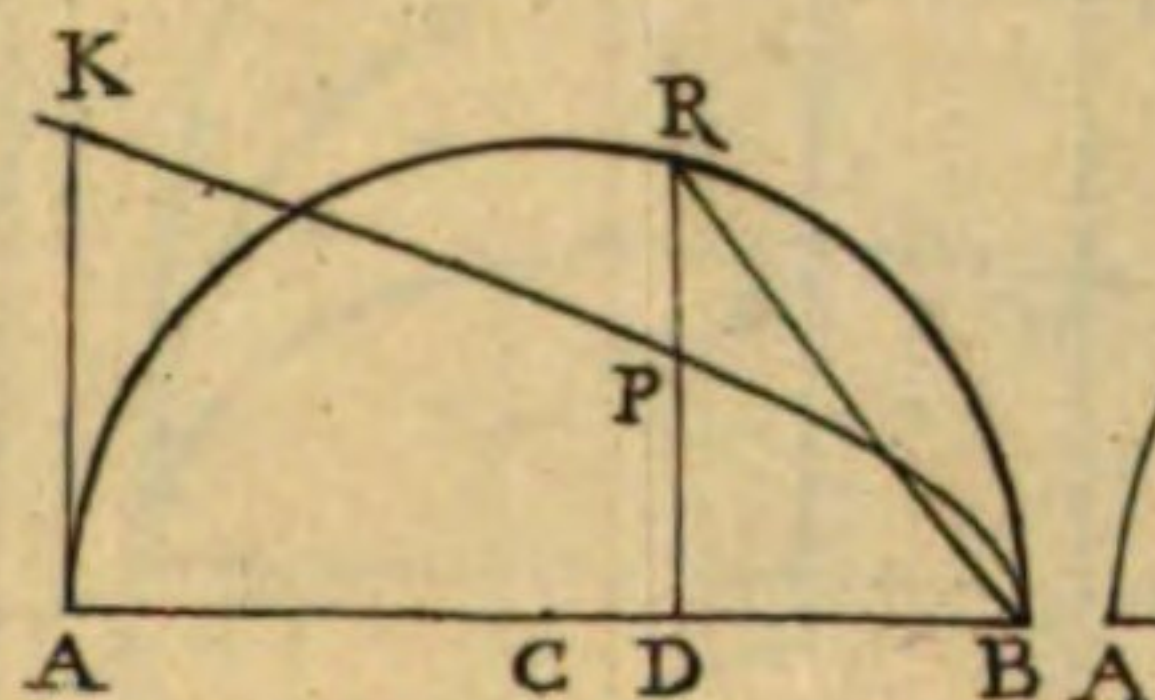


Fig. 2.

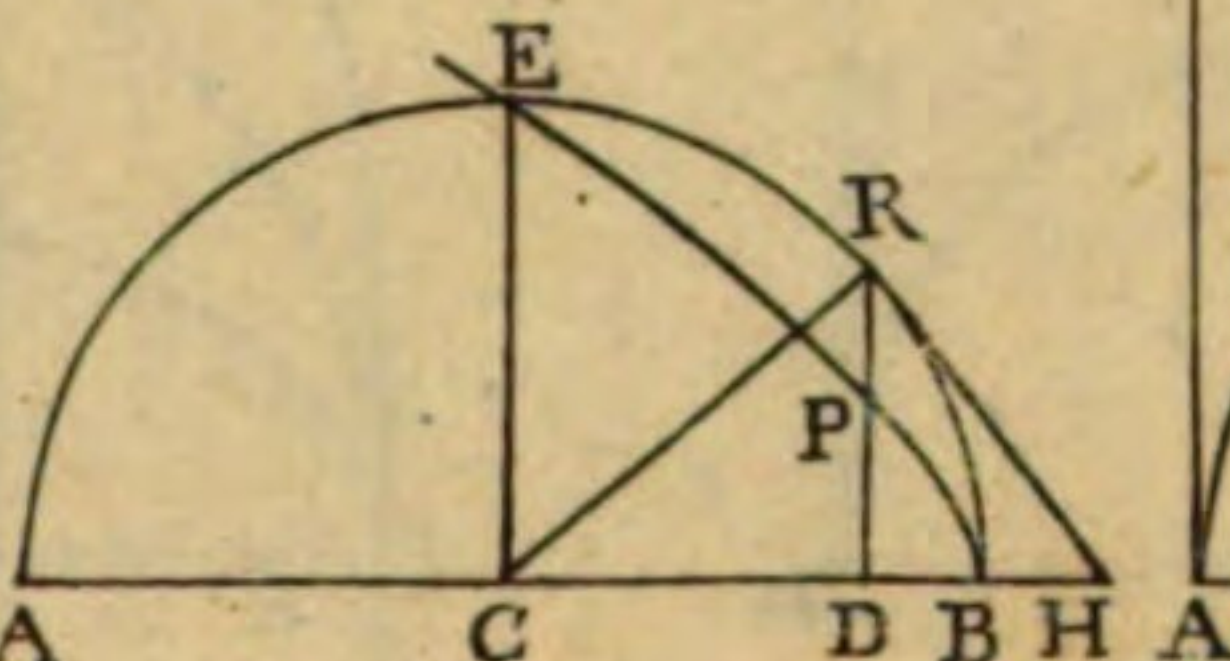


Fig. 3.

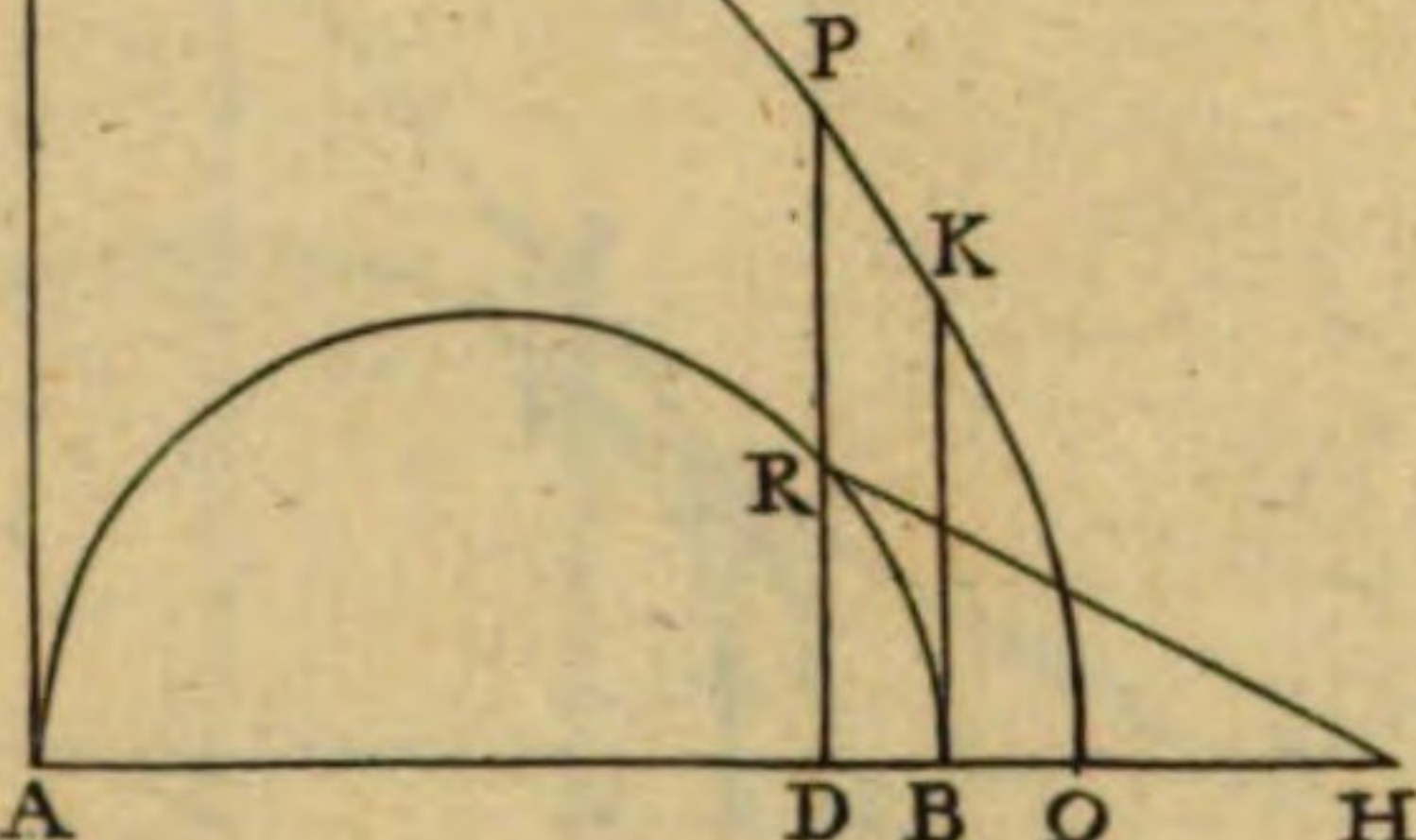


Fig. 4.

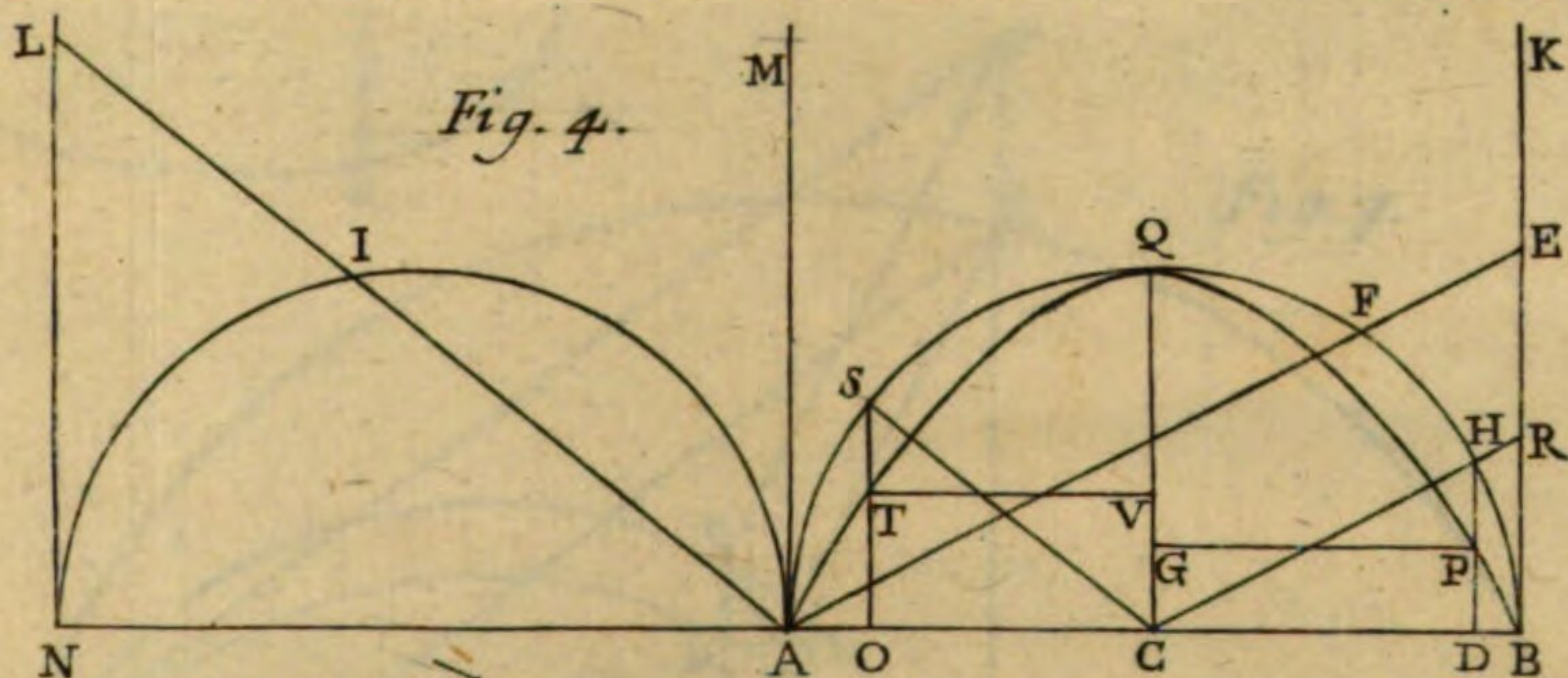
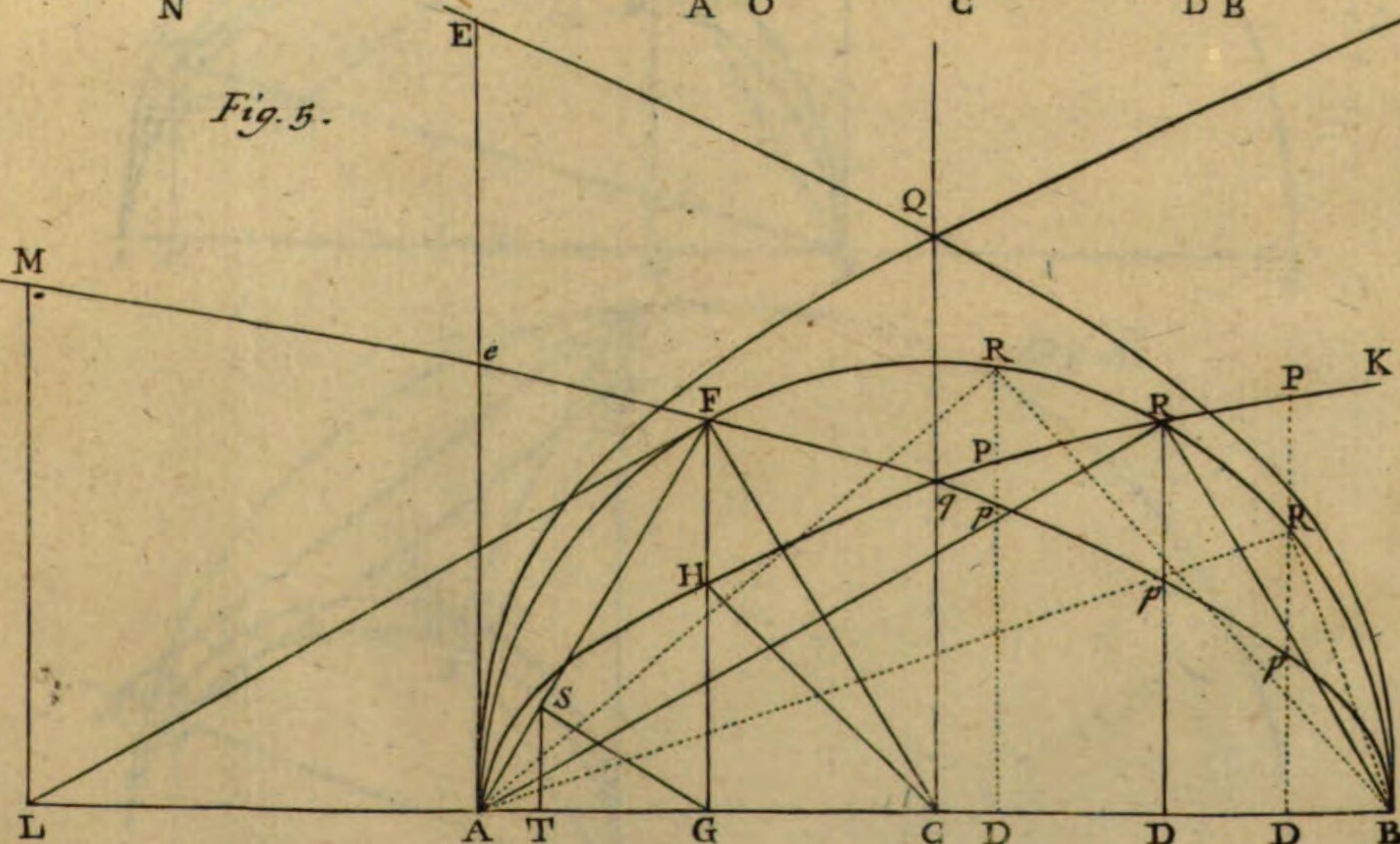


Fig. 5.



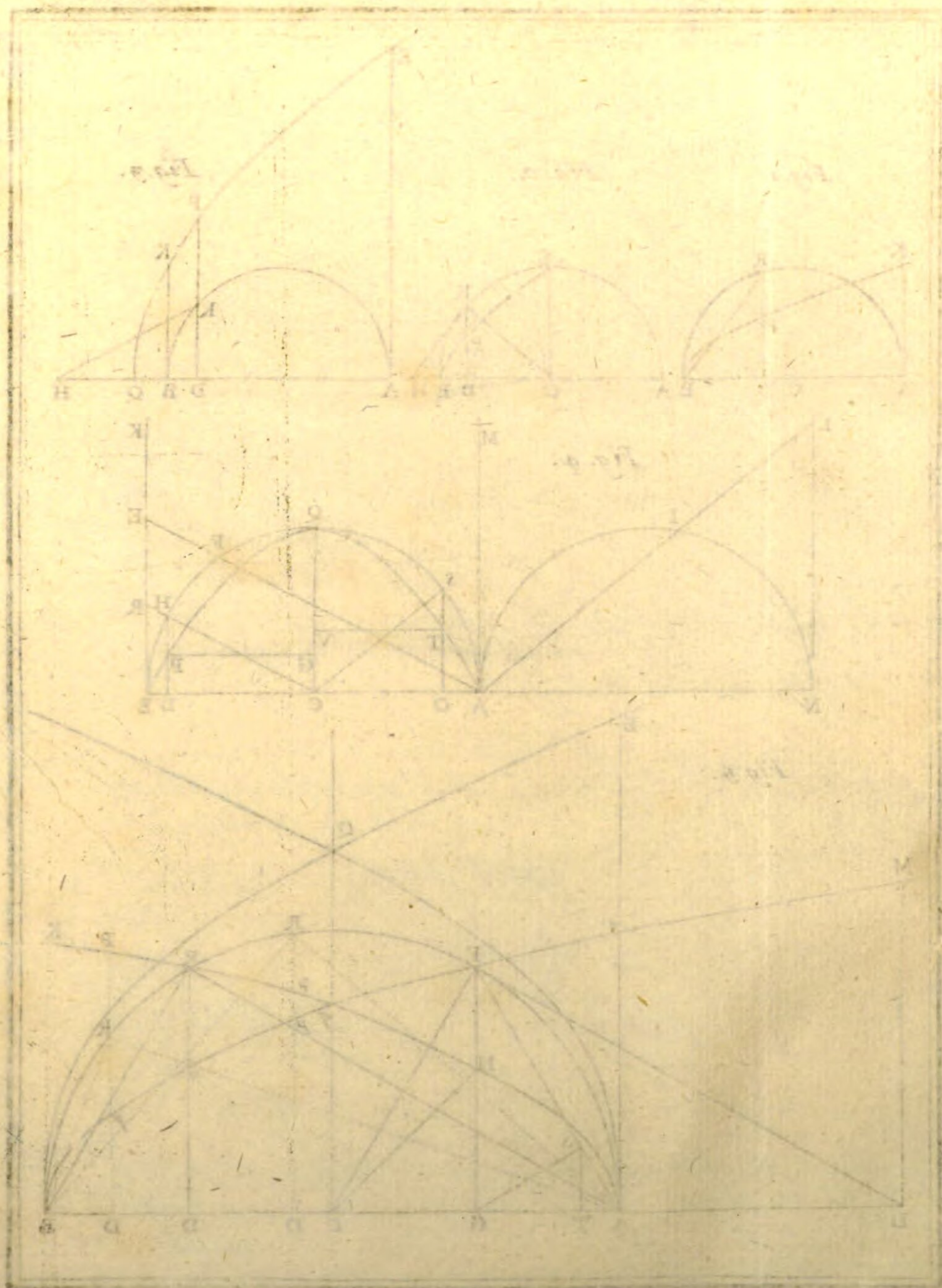


Fig. 6.

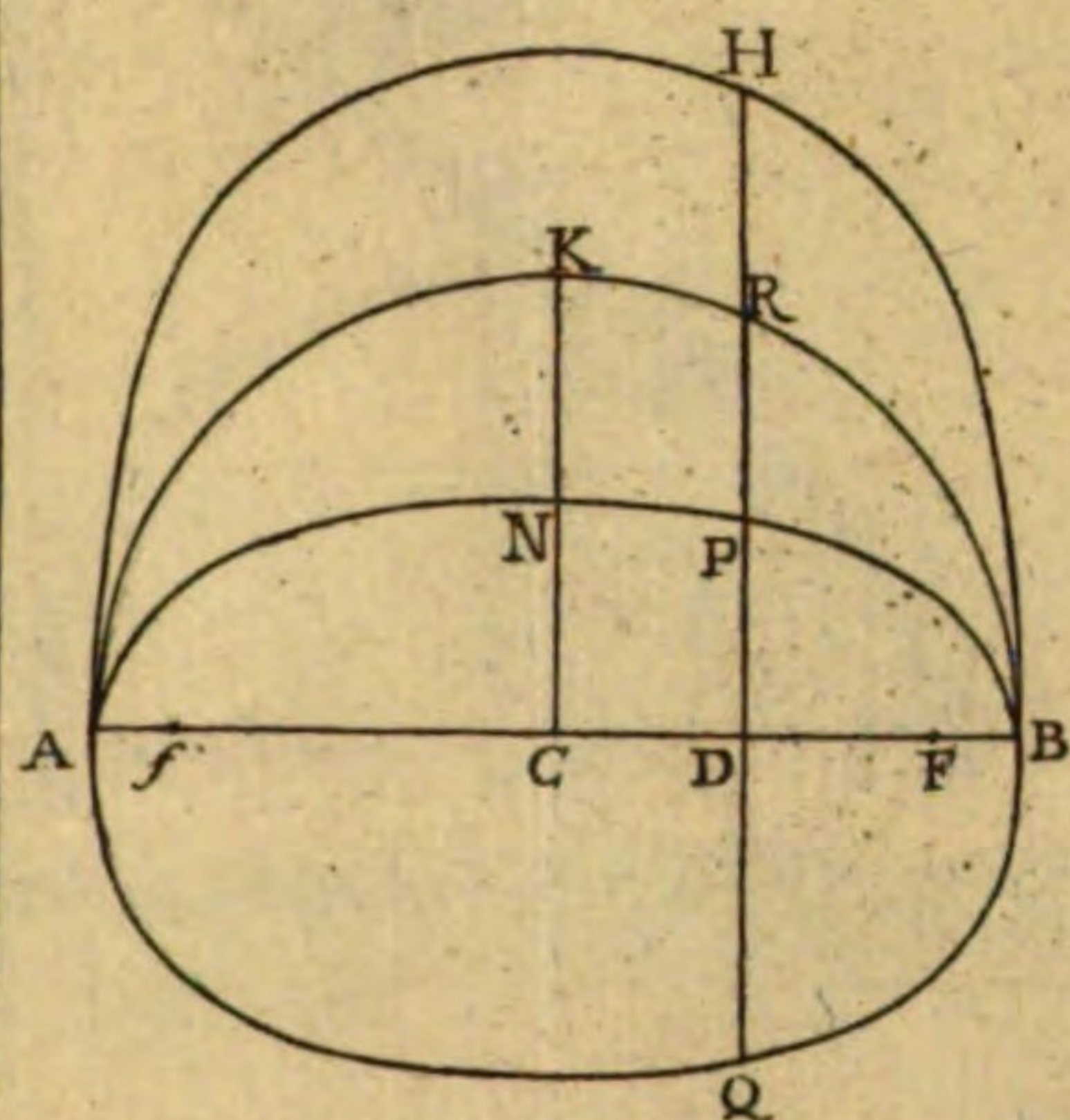


Fig. 9.

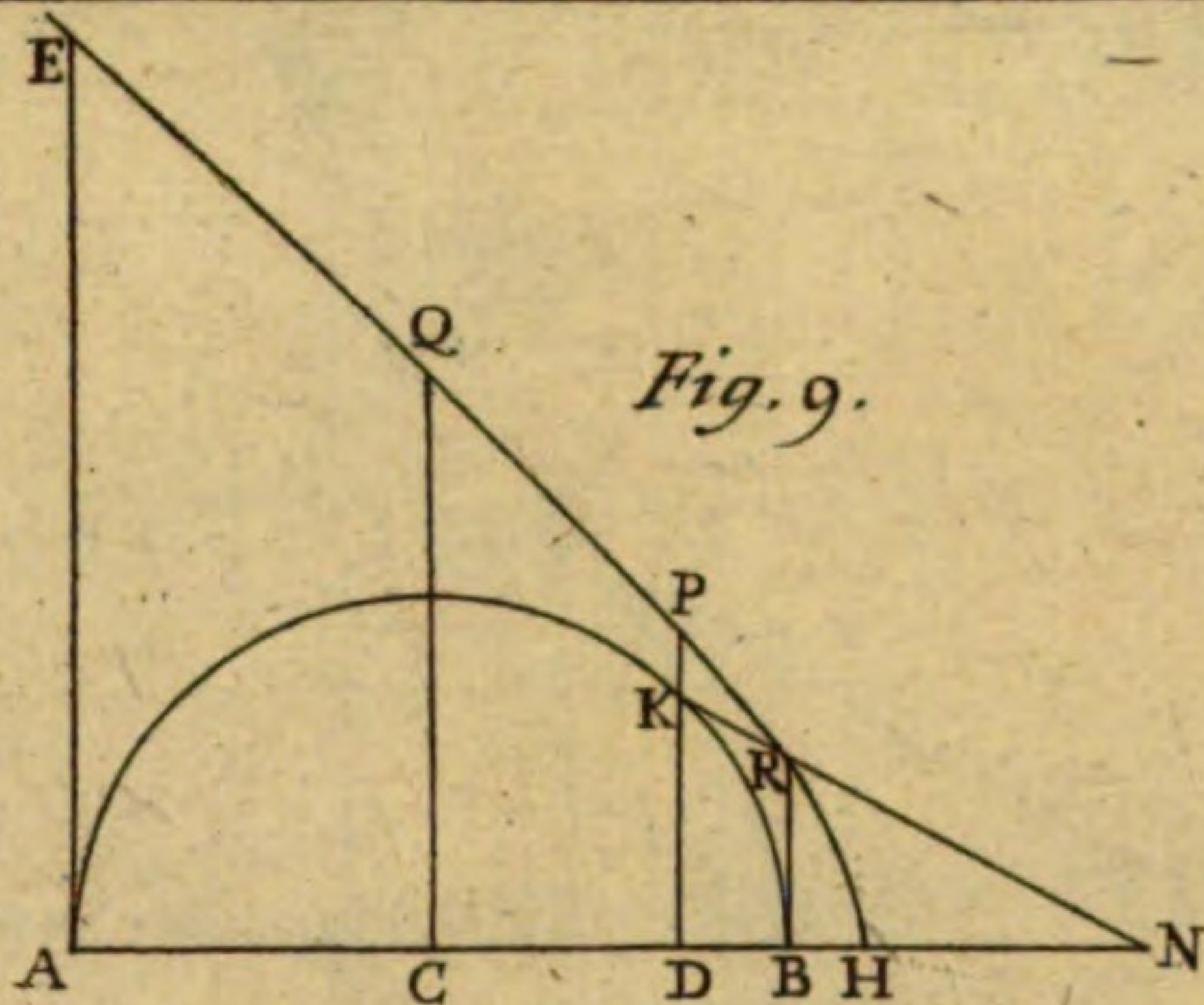


Fig. 7.

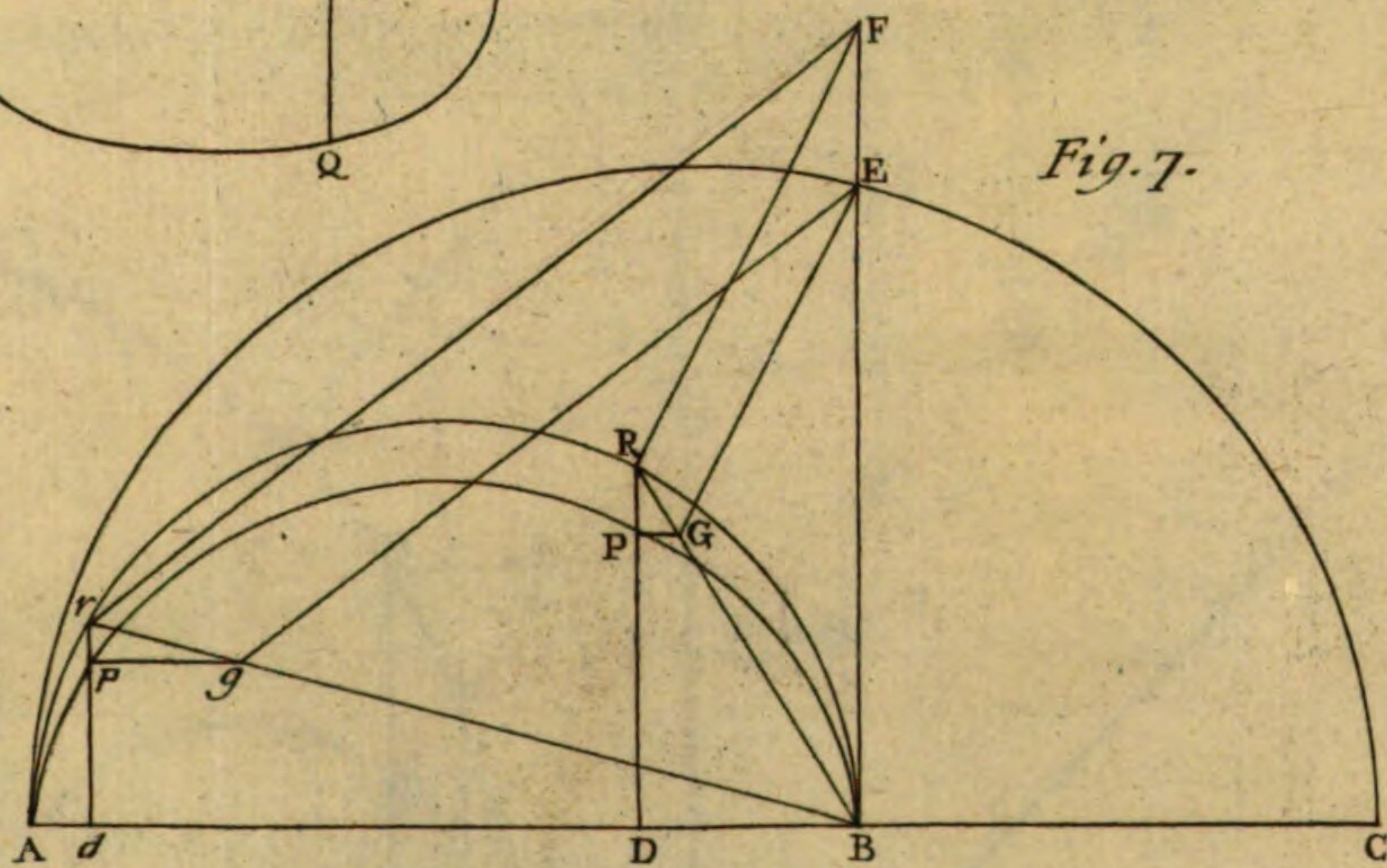


Fig. 8.

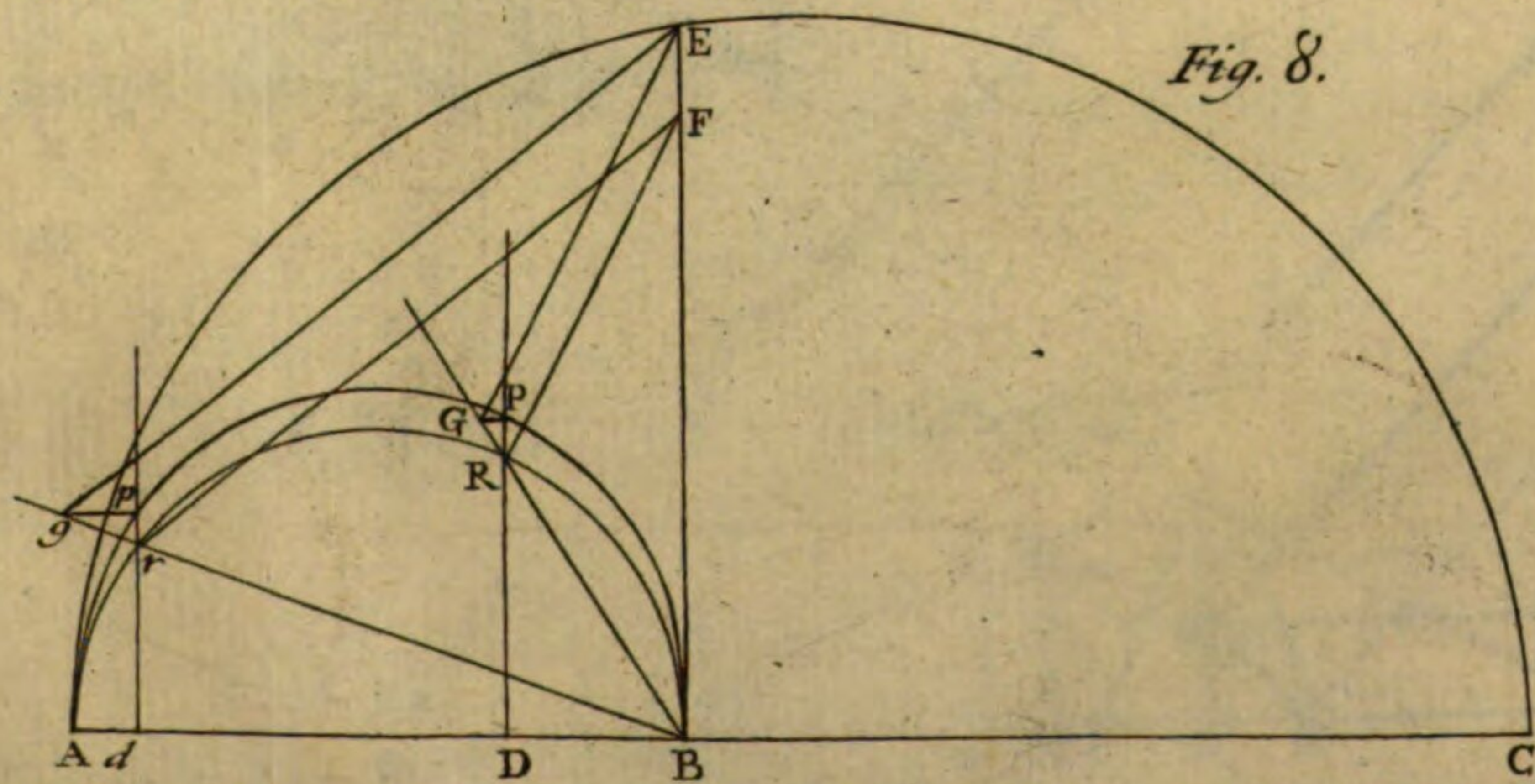


Fig. 6.

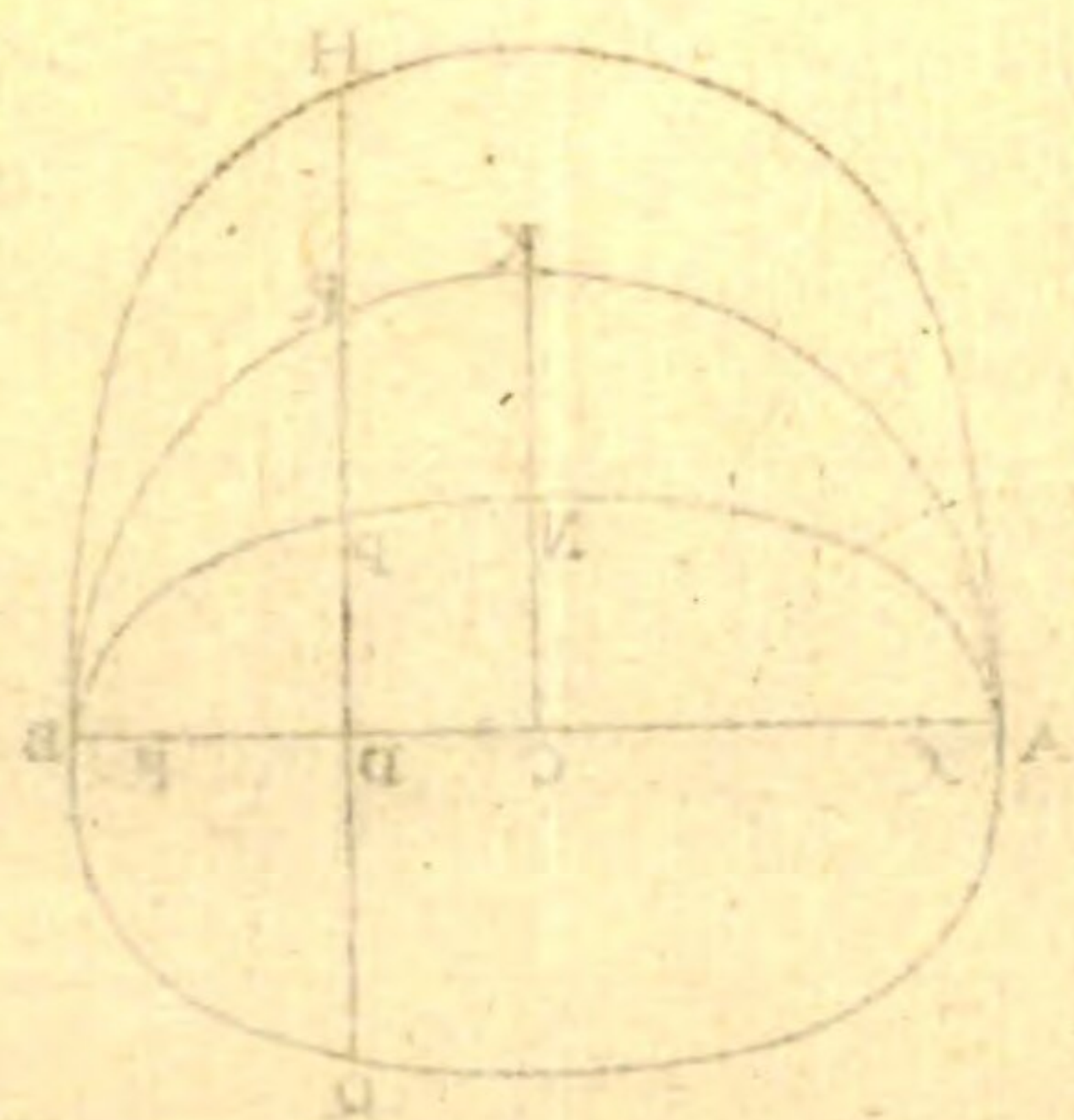


Fig. 7.

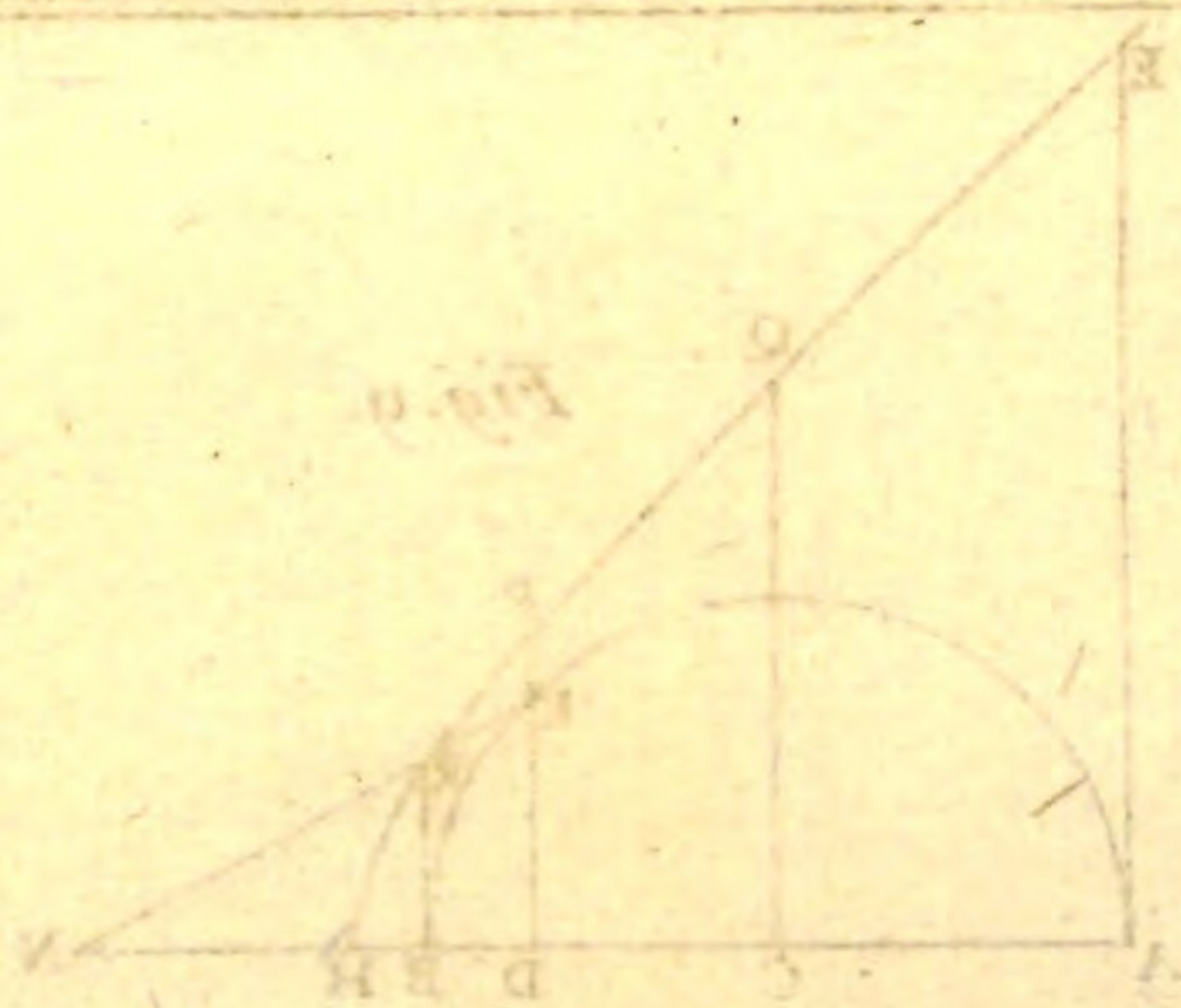


Fig. 1.

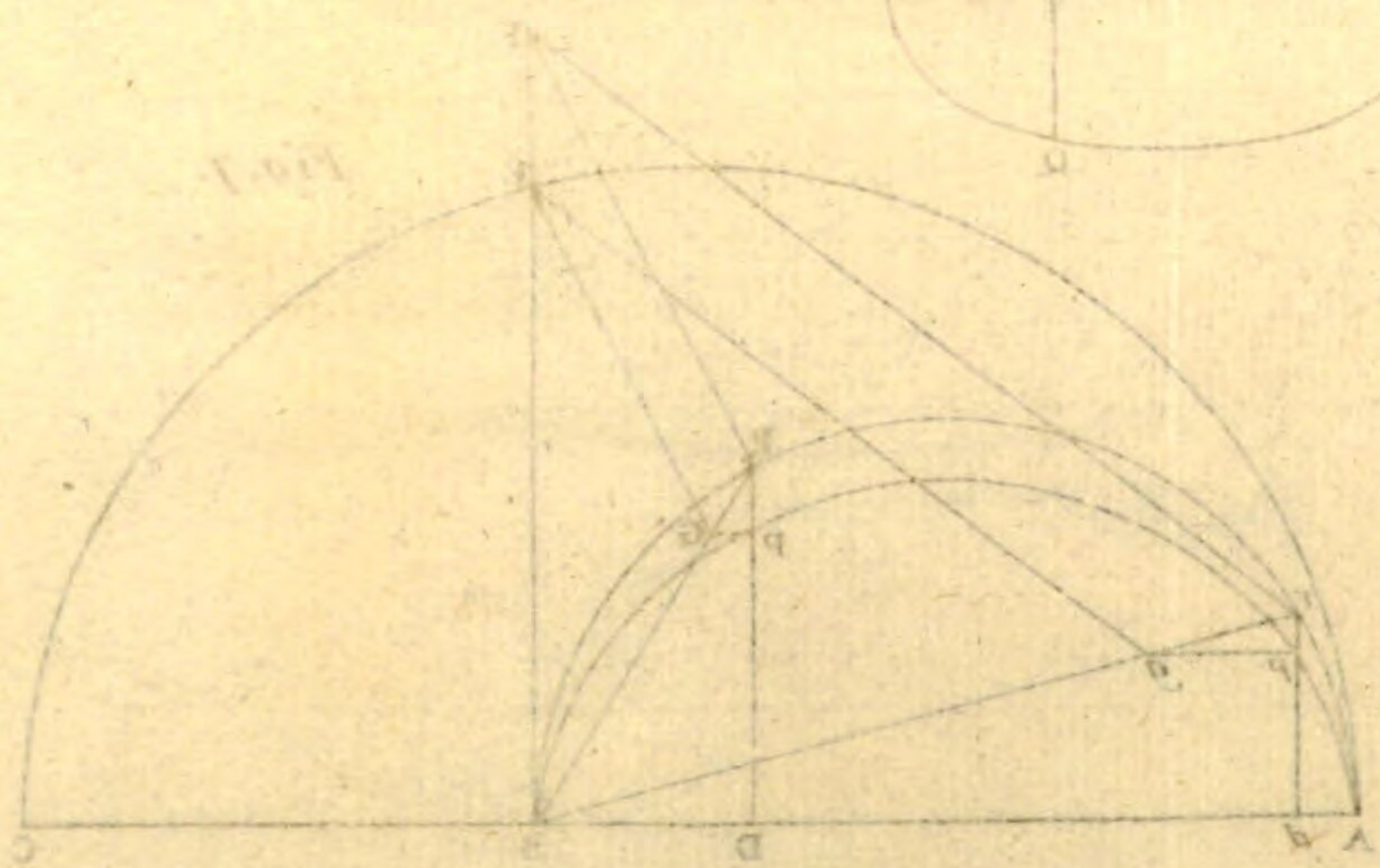
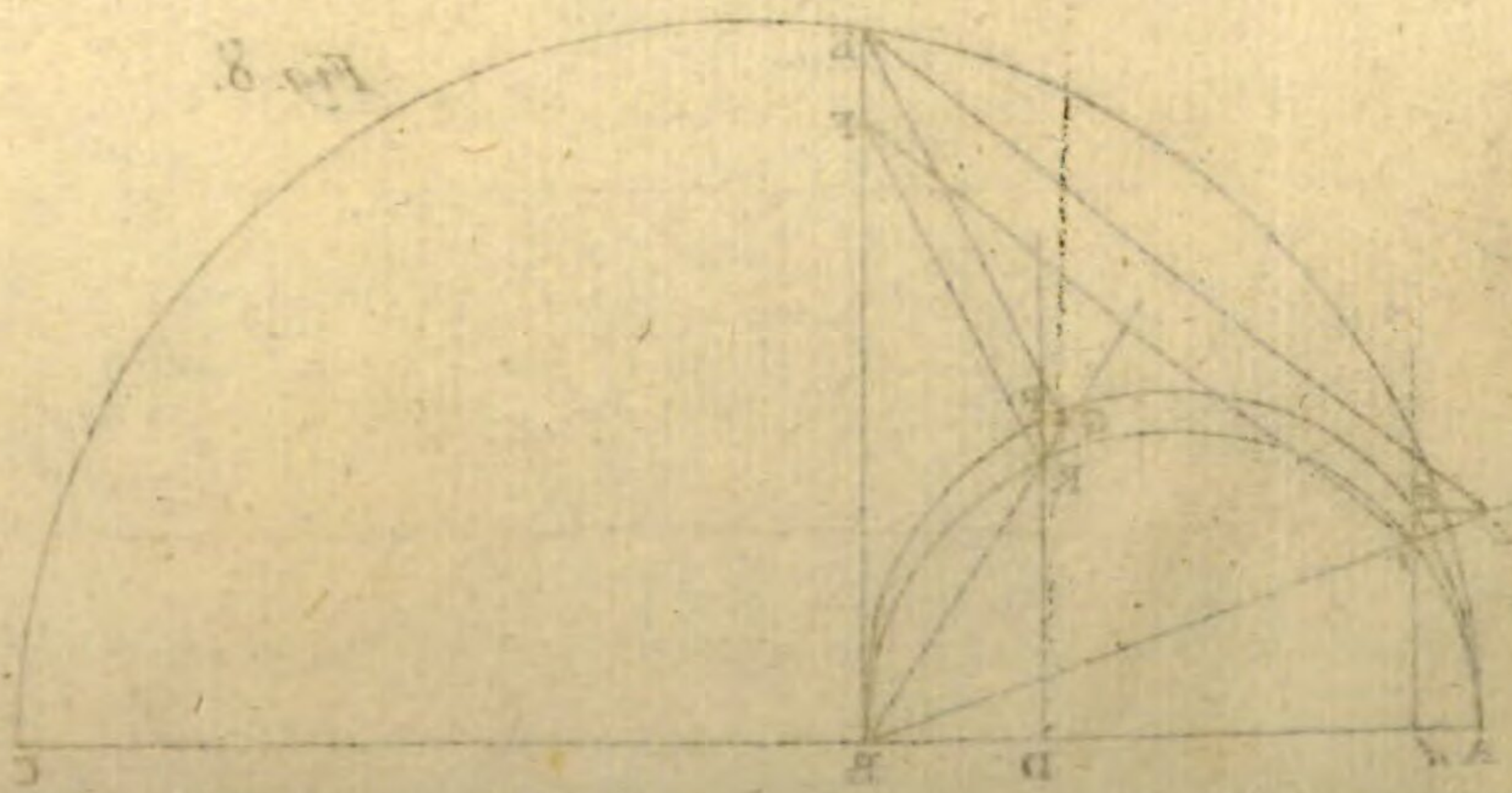


Fig. 8.



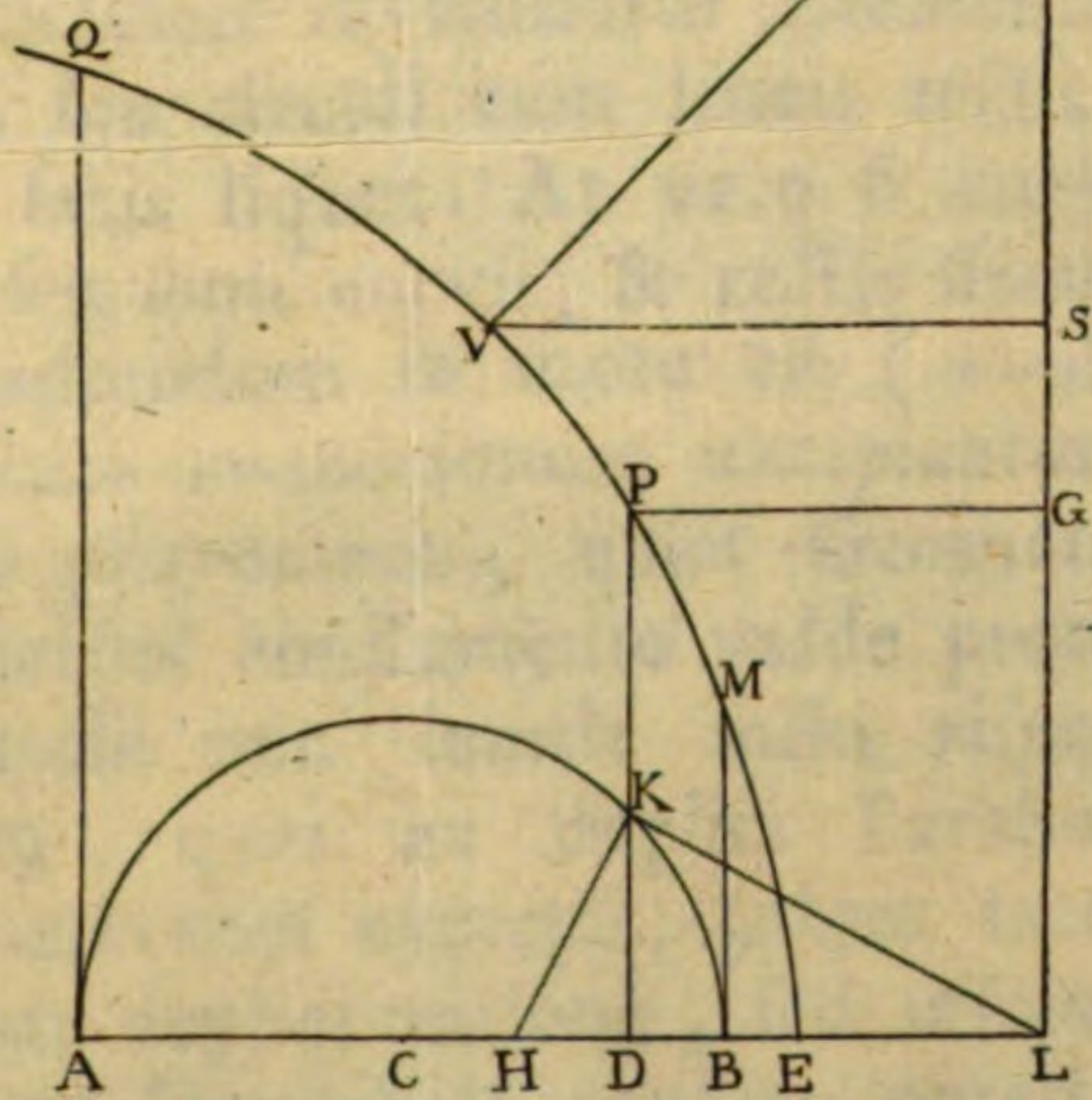
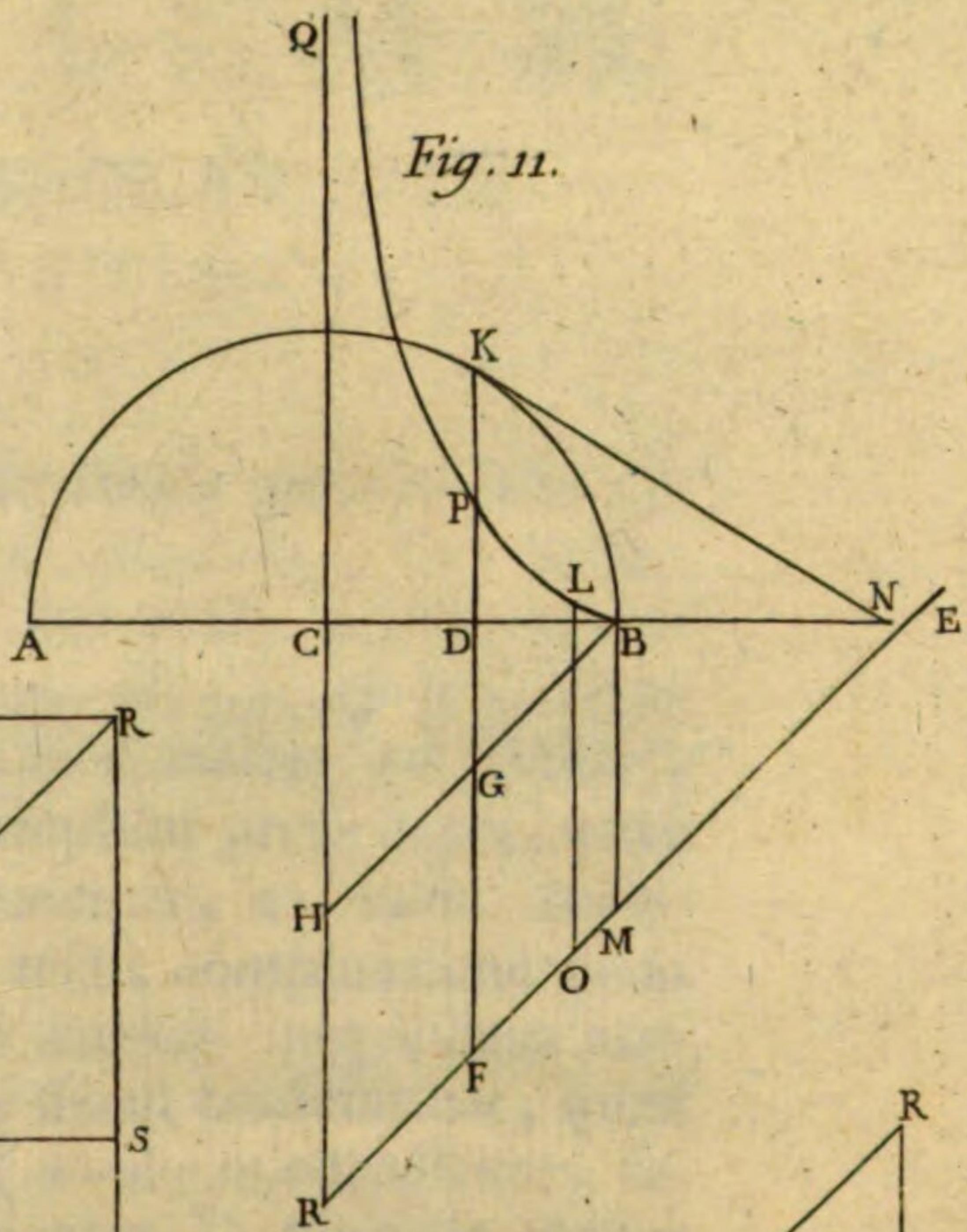
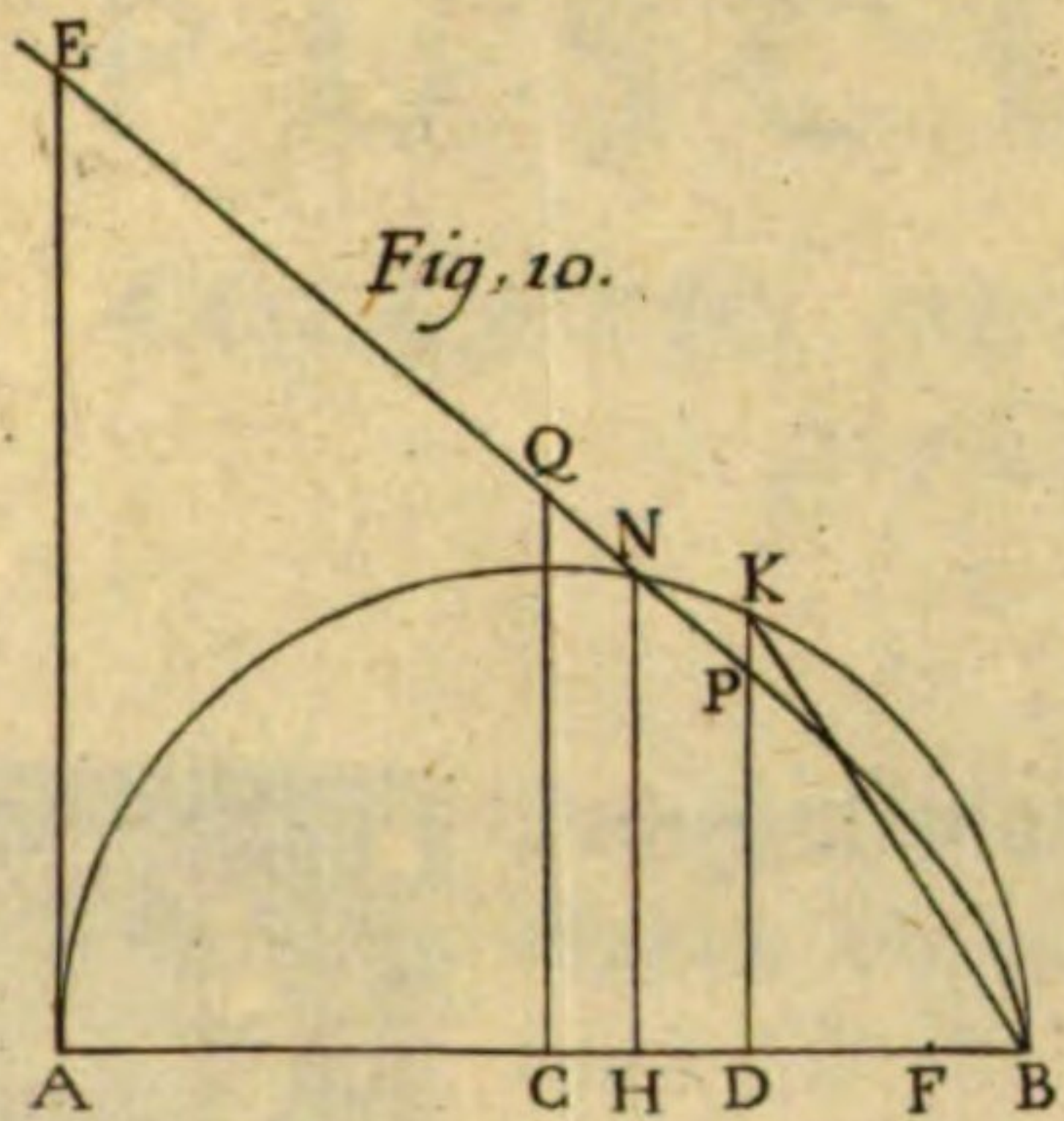
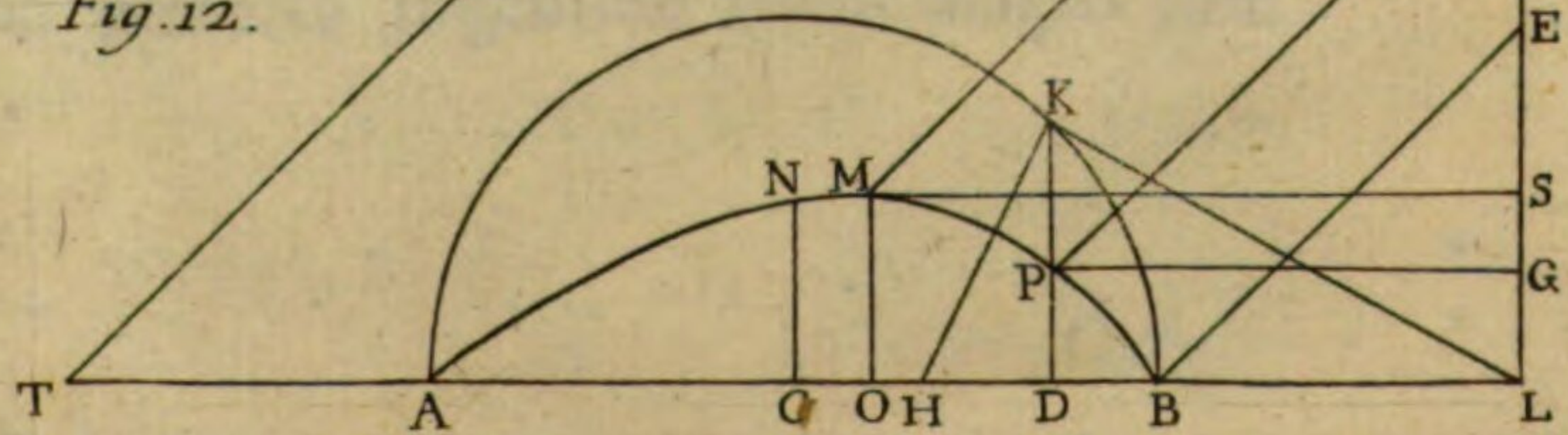
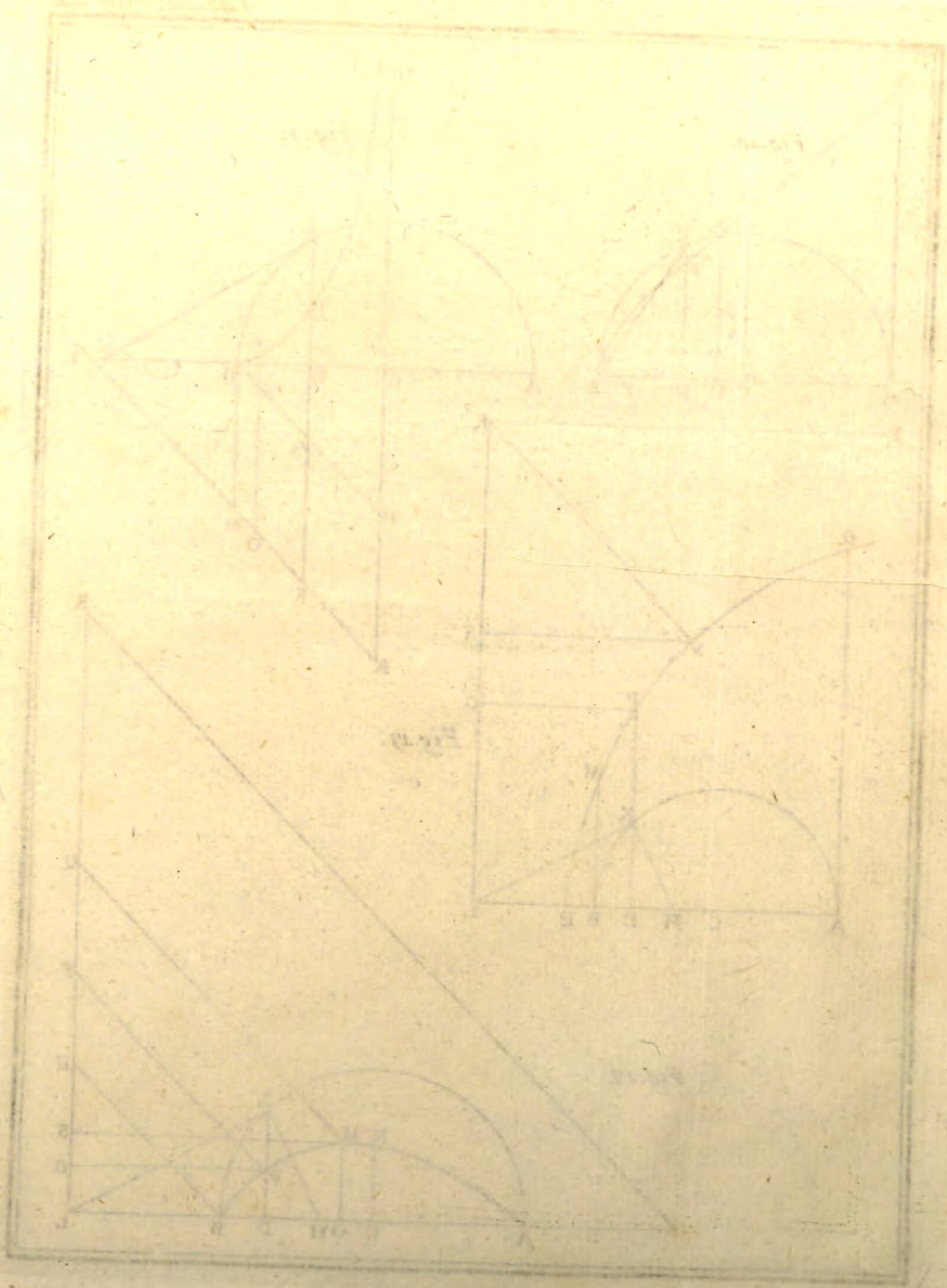


Fig. 13.

Fig. 12.





OPUSCULUM II.

COLLATIO PARABOLAE CUM HYPERBOLA.



Uamplurimas veritates geometricas ex mutua linearum rectarum, sive rectarum simul, & curvarum collatione repertas esse, nemo est qui ignoret. Proprietates enim fere omnes ad rectas, polygona, & circulum pertinentes, quae in Euclideis inveniuntur Elementis, ex varia rectarum, seu circuli cum lineis rectis combinatione ortas esse, satis liquet. At vero si curvae cum iisdem curvis, seu cum curvis, & rectis simul conferantur, quod non admodum in more est (modo constructiones superiorum aequationum excipiantur), plurima absque dubio provenient, quae Geometriae limites magno Matheſeos emolumento valde protendent. Hac itaque de causa non inutile mihi visum est quasdam veritates, quas ex duplici Parabolae, & Hyperbolae combinatione excerpſi, palam facere. Quoniam vero nedum duplici ratione, sed innumeris modis Parabola cum Hyperbola conferri potest, infinitae pene proprietates ex sola praedictarum curvarum comparatione emergent. Cuique igitur Mathematicarum rerum Indagatori haec sublimioris Geometriae jucunda pars objicitur, in qua profecto ingenium suum magna cum laude exercebit.

PRIMA PARABOLAE CUM HYPERBOLA
COLLATIO.

§. I.

Fig. 14. **S**it HNK quaelibet Hyperbola Apolloniana, & aequilatera, cujus asymptota AB, AO, & ANQ Parabola Apolloniana per verticem Hyperbolae N transiens, cujus apex A; quadratum cujuscunque abscissae AC erit ad quadratum respondentis semiordinatae CE, ut parameter Parabolae ad applicatam Hyperbolae CD.

Quoniam Parabola transit per verticem Hyperbolae N, erit $NM = MA$; consequenter MN erit parameter Parabolae. Quare $AC : CE :: CE : MN$, & $AC : CE :: CE : MN$. At vero $CE : MN :: AC : AM$. Igitur $AC : CE :: AC : AM$. Sed est quoque $AC : AM :: MN : CD$; hinc AC ad CE, ut MN, five parameter Parabolae, idest latus potentiae Hyperbolae, ad CD.

§. II.

Quia igitur supra verticem Hyperbolae N applicata CD semiordinatam Parabolae MN excedit, infra vero eundem verticem applicata *cd* ab ipsamet MN exceditur, hoc consequitur Theorema: si AN fuerit Parabola aequicruris, supra punctum M abscissae a semiordinatis exceduntur, infra vero idem punctum M semiordinatas excedunt.

§. III.

§. III.

Deducitur quoque ex §. I. nihilum Geometricum multiplex existere, seu quodcunque nihilum nequa-

quam ejusdem generis esse. Etenim $\overline{AC}^2 : \overline{CE}^2 :: MN :$

CD , & , posita abscissa $AC = 0$, $0 : \overline{CE}^2 :: MN :$

CD . At vero existente abscissa $AC = 0$, etiam semiordinata $CE = 0$. Quare $0 : 0 :: MN : CD$. Sed

cum $AC = 0$, applicata $CD = \infty$. Ergo MN erit pars infinitesima quantitatis CD , consequenter ob

analogiam etiam nihilum, quod ex AC emergit, erit infinitesimum alterius ex CE provenientis. Idem

dicendum de quantitatibus infinitis: etenim si ponatur $Ac = \infty$, erit $\infty : \infty :: MN : cd$; sed cd est

quantitas infinitesima relate ad MN ; ergo infinitum ex ce exurgens erit infinitesimum respectu alterius ex Ac emergentis.

§. IV.

Tandem quodlibet punctum ad Hyperbolam HK Fig. 15. pertinens, cujus latus potentiae aequatur parametro Parabolae AQ , ope ejusdem §. I. hac ratione reperitur. Quae-ratur ex. gr. applicata Hyperbolae CD , seu punctum Hyperbolae D , cujus abscissa AC . Ex puncto A agatur AE , cujus normalis sit EG , & accipiatur AM aequalis lateri potentiae Hyperbolae, agantur-que parallelae CM , GD ; erit punctum D in Hyperbola KH . Est enim CE media proportionalis inter AC , & CG ; adeoque AC ad CG erit in ratione duplicata rationis AC ad CE , idest $AC :$

$CG :: \overline{AC}^2 : \overline{CE}^2$. At vero est quoque $AM : CD ::$

$AC : CG$. Quare $AM : CD :: \overline{AC}^2 : \overline{CE}^2$. Sed AM aequa-

aequatur lateri potentiae Hyperbolae, seu parametro
Parabolae. Ergo CD est applicata Hyperbolae HK .

§. V.

Fig. 14. Quodlibet solidum genitum ex quadrato semiordi-
natae Parabolae CE in respondentem Hyperbolae ap-
plicatam CD aequatur cubo lateris potentiae MN .

Quoniam ex proprietate Parabolae $\overline{MN}^2 : \overline{CE}^2 ::$
 $AM : AC$, & ex alia Hyperbolae $AM : AC :: CD :$
 MN ; erit quoque $\overline{MN}^2 : \overline{CE}^2 :: CD : MN$. Quare
 $\overline{MN}^3 = \overline{CE}^2 \times CD$.

§. VI.

Quapropter omnia parallelepipeda praefata ratione
constructa erunt aequalia inter se, nempe quodlibet

quadratum $\overline{CE}^2$ ductum in applicatam CD efficiet pa-
rallelepipedum $\overline{CE}^2 \times CD$ aequale cuicunque alteri hoc
pacto genito; quandoquidem eorum quodlibet cubo
 $\overline{MN}^3$ aequatur: hinc quadrata semiordinatarum Pa-
rabolae CE , & erunt in ratione reciproca responden-
tium applicatarum Hyperbolae CD , *cd*.

§. VII.

Fig. 16. Sit modo Hyperbola HNK cubica; quodlibet re-
ctangulum ortum ex semiordinata CE in parametrum
Parabolae MN aequabitur rectangulo ex abscissa AC
in respondentem Hyperbolae applicatam CD .

Quia

Quia quadratum semiordinatae CE aequatur re-
ctangulo ex AC in MN , & ex proprietate Hyper-
bolae cubicae $\overline{MN}^3 = AC \times \overline{CD}^2$; erit multiplicando
inter se aequationes $\overline{CE}^2 \times \overline{MN}^3 = \overline{AC}^2 \times \overline{CD}^2 \times MN$,
seu $\overline{CE}^2 \times \overline{MN}^2 = \overline{AC}^2 \times \overline{CD}^2$, & extrahendo radicem
quadratam ex utroque aequationis membro $CE \times MN$
 $= AC \times CD$; ex qua sequens deducitur analogia AC :
 $CE :: MN : CD$.

§. VIII.

Quodlibet rectangulum ex respondentibus applicatis
 CE , CD genitum est aequale quadrato MN .

Quia Hyperbola est cubica, erit $\overline{CD}^2 : \overline{MN}^2 :: MA$:
 CA . At vero est quoque $MA : CA :: \overline{MN}^2 : \overline{CE}^2$.
Quare $\overline{CD}^2 : \overline{MN}^2 :: \overline{MN}^2 : \overline{CE}^2$, & $CD : MN :: MN$:
 CE , consequenter $\overline{MN}^2 = CD \times CE$.

§. IX.

Omnia igitur rectangula orta ex CD in CE erunt
aequalia inter se; & MN erit media proportionalis
inter quamlibet semiordinatam CE , & respondentem
applicatam Hyperbolae CD .

§. X.

Quod si Ac fuerit infinita, erit rectangulum $ec \times cd$
ortum ex infinita semiordinata ec , atque ex alia cd
D 2 infi.

infinitesima. Sed praefatum rectangulum aequatur quadrato MN . Quare quodlibet productum ex quantitate infinitesima in aliam infinitam (si tamen quantitas infinitesima est secundi ordinis relate ad infinitam, ut in praesenti casu) aequatur quantitati finitae. Idem dicendum, si AC fuerit infinitesima.

§. XI.

Fig. 17. Habetur ex §. VIII. mutua descriptio Hyperbolae cubicae ope Parabolae, & vicissim Parabolae ope Hyperbolae. Sit curva BAE Parabola Apolloniana, & CE , seu CB quaelibet semiordinata. Accipiatur recta CD , quae sit tertia proportionalis post applicatam CE , & parametrum Parabolae; punctum D erit in Hyperbola cubica DNd , cuius apex erit punctum N ; quandoquidem rectangulum ex CE in CD erit aequale quadrato parametri Parabolae, idest MN . Quod si fuerit descripta Hyperbola cubica DNd , sumatur CB tertia proportionalis ad CD , & MN ; punctum B erit in Parabola AB , cuius parameter MN ; erit enim factum ex BC , seu CE in CD aequale eidem quadrato MN .

§. XII.

Quoniam quodlibet rectangulum ex BC in respondentem applicatam CD aequatur cuilibet alteri tali ratione constructo, hoc consequitur Theorema: semiorinatae Parabolae BC , bc sunt in ratione reciproca respondentium applicatarum Hyperbolae cubicae CD , cd .

§. XIII.

§. XIII.

Sit CD quaelibet applicata Hyperbolae cubicae, cui semiordinata Parabolae CE respondet; rectangulum ex DE in EC erit aequale rectangulo ex MN in MC .

Sit primo applicata CD supra verticem Hyperbolae N . Erit itaque $CE \times CD = \overline{MN}^2$ (§. VIII.), seu $\overline{MN}^2 = CE \times (CE + ED) = \overline{CE}^2 + CE \times ED$.

Quare erit $CE \times ED = \overline{MN}^2 - \overline{CE}^2$. At vero $\overline{CE}^2 = AC \times MN$. Igitur $CE \times ED = \overline{MN}^2 - AC \times MN = MN \times (MN - AC) = MN \times MC$.

Sit modo cd infra verticem N . Quoniam $ce = cA \times MN$, seu $cA \times MN = ce \times (cd + de) = ce \times cd + ce \times de$; erit $ce \times de = cA \times MN - ce \times cd$. Sed $ce \times cd = \overline{MN}^2$. Ergo $ce \times de = cA \times MN - \overline{MN}^2 = MN \times (cA - MN) = MN \times cM$.

§. XIV.

Protrahatur NM in Q , & agatur DP parallela axi AM , quae occurrat lateri MN producto, si opus est; dico rectangulum ex CD in DE esse aequale rectangulo ex QP in PN .

Sit CD supra apicem N . Quoniam $\overline{MN}^2 = CD \times CE$ (§. VIII.), & $\overline{CD}^2 = DC \times CE + CD \times DE$, seu $DC \times CE = \overline{CD}^2 - CD \times DE$; erit $\overline{MN}^2 = \overline{CD}^2 - CD \times DE$. Sed $CD = MP$; igitur $\overline{MN}^2 = \overline{MP}^2 - MP \times DE$.

$$— CD \times DE, \& \overline{CD} \times \overline{DE} = \overline{MP} — \overline{MN} = QP \times PN.$$

Sit nunc cd infra verticem N . Quia $\overline{MN} = \overline{ec} \times \overline{cd} = \overline{cd} \times (\overline{cd} + \overline{de}) = \overline{cd} + \overline{cd} \times \overline{de}$; erit $\overline{cd} \times \overline{de} = \overline{MN} - \overline{cd} = \overline{MP}$. At vero $\overline{cd} = \overline{Mp}$; ergo $\overline{cd} \times \overline{de} = \overline{MN} - \overline{Mp} = QP \times PN$.

§. XV.

Fig. 18. Si ex quolibet puncto D Hyperbolae cubicae agatur DQ normalis applicatae DC , quae Parabolae in puncto Q occurrat, & compleatur rectangulum QC ; erit applicata Hyperbolae KB aequalis semiordinatae Parabolae EC .

Nam cum sit rectangulum ex DC in CE aequale rectangulo ex QB in BK (§. IX.), & $CD = BQ$; erit quoque $CE = BK$.

§. XVI.

Quapropter si jungantur puncta K, E recta KE , spatium $CEKB$ erit rectangulum. Et quoniam $CE = BK$, & $CD = BQ$; erit etiam $ED = KQ$, & consequenter rectangulum ex DE in EC erit aequale rectangulo ex QK in KB .

§. XVII.

Rectae AC, AM , & AB sunt in continua proportionem geometrica.

Est enim recta CE aequalis rectae BK (§. XV.);
hinc

hinc $\overline{CD} : \overline{MN} :: \overline{MN} : \overline{BK}$ (§. IX.). At vero
 $\overline{CD} : \overline{MN} :: \overline{MA} : \overline{CA}$, & $\overline{MN} : \overline{BK} :: \overline{BA} : \overline{MA}$.
 Igitur $\overline{MA} : \overline{CA} :: \overline{BA} : \overline{MA}$, & invertendo $\overline{CA} : \overline{MA} :: \overline{MA} : \overline{BA}$.

§. XVIII.

Rectangulum ex $\overline{DC}$ in $\overline{CE}$ aequatur rectangulo ex $\overline{BA}$ in $\overline{AC}$.

Quadratum $\overline{MN}$ aequatur rectangulo ex $\overline{DC}$ in $\overline{CE}$ (§. VIII.); pariter idem quadratum $\overline{MN}$ est aequale facto ex $\overline{BA}$ in $\overline{AC}$ (§. XVII.). Quare $\overline{DC} \times \overline{CE} = \overline{BA} \times \overline{AC}$; hinc $\overline{AC} : \overline{CE} :: \overline{DC} : \overline{BA}$.

§. XIX.

Producatur $\overline{DC}$ in $\overline{F}$, & agatur $\overline{FG}$ parallela axi $\overline{AB}$, quae occurrat lateri $\overline{MN}$ protracto; dico rectangulum ex $\overline{DF}$ in $\overline{FG}$, seu $\overline{FP}$ aequari rectangulo $\overline{EB}$.

Quoniam tres rectae $\overline{AB}$, $\overline{AM}$, & $\overline{AC}$ sunt in continua proportionem geometrica (§. XVII.); erit dividendo, & permutando $\overline{BM} : \overline{CM} :: \overline{AM} : \overline{AC}$. Sed

$\overline{AM} : \overline{AC} :: \overline{MN} : \overline{CE}$. Igitur erit quoque $\overline{BM} : \overline{CM} :: \overline{MN} : \overline{CE}$. At vero $\overline{MN} : \overline{CE} :: \overline{CD} : \overline{CE}$, quia $\overline{MN}$ est media proportionalis inter $\overline{CD}$, & $\overline{CE}$ (§. IX.). Quare $\overline{BM} : \overline{CM} :: \overline{CD} : \overline{CE}$; & consequenter erit rectangulum ex $\overline{BM}$ in $\overline{CE}$, seu $\overline{MK}$ aequale rectangulo ex $\overline{CM}$ in $\overline{CD}$, idest $\overline{CP}$. Sed etiam rectangulum $\overline{EM}$ aequatur rectangulo $\overline{MF}$. Ergo $\overline{EC} \times \overline{CB} = \overline{DF} \times \overline{FG}$.

Eadem

Eadem est demonstratio, si applicata dc fuerit infra apicem N ; nisi quod loco rectarum continue proportionalium AB , AM , & AC sumendae sunt Ac , AM , & Ab , quae pariter sunt in continua proportionem geometrica.

§. XX.

Fig. 16. Parallelepipedum ortum ex quadrato abscissae AC in respondentem applicatam Hyperbolae CD aequatur cubo semiordinatae Parabolae CE .

Quia CE est media proportionalis inter AC , & MN , erit $\overline{AC}^2 : \overline{CE}^2 :: \overline{CE}^2 : \overline{MN}^2$. At vero $\overline{MN}^2 = DC \times CE$. Igitur $\overline{AC}^2 : \overline{CE}^2 :: \overline{CE}^2 : DC \times CE$, consequenter $\overline{AC}^2 : \overline{CE}^2 :: CE : DC$, & $\overline{CE}^3 = \overline{AC}^2 \times DC$.

§. XXI.

Quamobrem quadratum abscissae AC erit ad quadratum semiordinatae CE , ut eadem applicata CE ad respondentem ordinatam Hyperbolae CD ; & cubus semiordinatae Parabolae $\overline{CE}^3$ ad quemlibet alium $\overline{ce}^3$ in ratione composita quadratorum abscissarum AC , Ac , & respondentium Hyperbolae applicatarum CD , cd .

§. XXII.

Quodlibet spatium Hyperbolico-cubicum $CAOHDC$ interminatum aequatur duplo rectangulo ex parametro Parabolae MN in respondentem semiordinatam CE .

Nam

Nam praefatum spatium Hyperbolicum aequatur duplo cubo lateris MN per respondentem applicatam CD diviso, nempe $2 \overline{MN}^3 : CD$. At vero $\overline{MN}^2 = DC \times CE$. Quare erit $2 \overline{MN}^3 : CD = 2 MN \times DC \times CE : CD = 2 MN \times CE$; ideoque spatium $CAOHDC$ erit aequale duplo rectangulo ex MN in CE .

§. XXIII.

Quapropter spatium interminatum $MAOHNM$ erit aequale duplo quadrato parametri Parabolae MN , quandoquidem in puncto M semiordinata parametro aequatur. Quare si erigatur perpendiculum NP , spatium interminatum $NPOHN$ erit aequale quadrato AN . Alterum vero spatium $DROHD$ aequatur rectangulo AD ; nam $MN \times CE = AC \times CD$ (§. VII.), consequenter $2 MN \times CE = 2 AC \times CD$, & $AC \times CD = DROHD$.

§. XXIV.

Spatium Hyperbolico-cubicum $NPOHN$ interminatum est infinitesimum relate ad alterum spatium $MNKBM$ pariter interminatum.

Etenim spatium $NPOHN$ aequatur quadrato $\overline{MN}^2$ (§. XXIII.), & $BAOHKB$ duplo rectangulo ex BQ in MN (§. XXII.); consequenter spatium $MNKBM$ erit aequale duplo rectangulo ex BQ in MN duplo quadrato $\overline{MN}^2$ mulato, nempe $MNKBM = 2 BQ \times MN - 2 \overline{MN}^2$. Quapropter erit $MNKBM : NPOHN :: 2 BQ \times MN - 2 \overline{MN}^2 : \overline{MN}^2 :: BQ - E : MN$.

$MN : \frac{1}{2} MN$. At vero quantitas $\frac{1}{2} MN$ est infinitesima relate ad BQ — MN . Ergo etiam spatium $NPOHN$ erit infinitesimum respectu alterius $MNKBM$.

§. XXV.

Qui in Analyfi Infinitorum Tyrones adhuc sunt, contradictionem in §. §. X. XXIII. & XXIV. reperi-ri forsan autumabunt. Etenim in §. superiori demon- stravimus spatium $NPOHN$, nempe quadratum AN esse quantitatem infinitesimam relate ad spatium $MNKBM$; consequenter idem spatium $MNKBM$, seu $BKHOAB$ erit quantitas infinita. Quare du- plum rectangulum ex AB in BK erit quoque spa- tium infinitum (§. XXIII.), quod vi §. X. debet esse finitum, quia recta AB est infinita, BK vero infi- nitesima. At vero haec omnia diluuntur, si adver- tatur quantitatem BK non esse infinitesimam secun- di ordinis relate ad quantitatem AB , uti supponitur

in §. X., sed tertii ordinis. Enimvero $BQ^2 = AB \times MN$, ideoque $MN : BQ :: BQ : AB$. Ast MN est infinitesima respectu alterius BQ ; ergo etiam BQ erit infinitesima relate ad aliam AB , consequenter MN erit infinitesima secundi ordinis respectu AB . At vero BK est infinitesima relate ad MN . Igitur BK erit infinitesima tertii ordinis respectu quanti- tatis AB .

§. XXVI.

Ramus Hyperbolae cubicae NH proximior est as- ymptoto AO , quam ramus NK alteri AB .

Sumatur AG aequalis rectae AR , & agatur se- miordinata GL . Quoniam $AC \times CD = MN \times CE$,
& AG

& $AG \times GT = MN \times GL$ (§. VII.); erit $AC \times CD$:
 $AG \times GT :: MN \times CE$: $MN \times GL$. At vero $AC \times$
 CD : $AG \times GT :: AC$: $GT :: RD$: GT , & $MN \times$
 CE : $MN \times GL :: CE$: GL . Igitur RD : $GT ::$
 CE : GL . Sed semiordinata CE minor est alia GL .
 Ergo etiam RD minor erit GT ; proindeque ramus
 NH propinquior est asymptoto AO , quam NK al-
 teri AB .

§. XXVII.

Quare mirum non est, si spatium $MNKB$ est
 infinitum, alterum vero $NPOHN$ finitum, sicuti
 probavimus in §. XXIV., quamvis utrumque in infi-
 nitum sit productum: spatia enim $MNKB$, $NPOHN$
 sunt inaequalia, quia rami NK , NH inaequaliter ad
 asymptota AB , AO inclinantur. Hinc etiam dedu-
 citur, quod spatium Hyperbolicum FNH majus est
 altero FNK .

§. XXVIII.

Si sumatur MP aequalis trienti lateris MN , & ex *Fig. 19.*
 vertice Parabolae A per idem punctum P agatur re-
 cta AE usque ad Parabolam, ducaturque semiordina-
 ta EC ; erit spatium Parabolicum CAE triplum spa-
 tii Hyperbolico-cubici $CAOHND$ interminati.

Nam AM : $MP :: AC$: CE . Sed AM : $MP ::$
 3 : 1 ; igitur erit AC : $CE :: 3$: 1 ; ideoque $3CE$
 $= AC$. Quapropter spatium Parabolicum $CAE =$

$\frac{2}{3} AC \times CE = \frac{6}{3} CE^2 = 2 CE^2$. Spatium vero Hyper-
 bolicum $CAOHND = 2 MN \times CE$ (§. XXII.).

Quare spatium CAE : $CAOHND :: 2 CE^2$:
 $2 MN \times CE :: CE$: MN . At vero AC : $CE ::$
 E 2 CE :

$CE : MN :: AM : MP :: 3 : 1$. Ergo $CAE : CAOHNDC :: 3 : 1$, & $CAE = 3 CAOHNDC$.

§. XXIX.

Quamobrem quia AC est tripla rectae CE , CE tripla MN , & MN pariter tripla rectae MP , seu quia $AC : CE :: CE : MN$, & $CE : MN :: AM (=MN) : MP$; erit $AC : CE :: CE : MN :: MN : MP$. Quare quatuor rectae AC , CE , MN , & MP sunt in continua proportionem geometrica.

C A P U T II.

ALTERA PARABOLAE CUM HYPERBOLA
COMPARATIO.

§. I.

Fig. 20. **S**it SG tangens verticalis cujuslibet Hyperbolae Apolloniana, & scalenae KTH , cujus asymptota DR , DP angulum acutum RD comprehendant, sitque GF normalis asymptoto DR ; summa rectarum GD , DF erit tertia proportionalis ad latus potentiae Hyperbolae TE , & dimidium axem transversum DT .

Demittatur perpendicularum TY . Quoniam trigona GDF , TEY sunt similia, erit $GD + DF : GD :: TE + EY : TE$. Sed TE aequatur rectae ED . Igitur $GD + DF : GD :: DY : ED$, & permutando $GD + DF : DY :: GD : ED$; consequenter cum sit GD dupla rectae ED , erit summa rectarum GD , DF aequalis duplae rectae DY . Qua-

propter $DT^2 = GD \times DY = 2DE \times DY = DE \times 2DY$

37

$2DY = TE \times (GD + DF)$; proindeque TE, DT, GD + DF sunt in continua proportione geometrica.

§. II.

Si Hyperbola est aequilatera recta DF evanescit, quia angulus GDS rectus est; ideoque recta DG erit tertia proportionalis ad TE, & TD: id quod etiam patet ex vulgari Geometria.

§. III.

Quod si assymptota DR, DP angulum obtusum *Fig. 21* RDP comprehendant, demittatur GF, quae sit perpendicularis assymptoto RD producto; differentia rectarum GD, ac DF erit tertia proportionalis ad TE, & DT.

Quoniam anguli TEY, FDG sunt alterni, ideoque aequales, erunt trigona TEY, GDF similia; consequenter $TE - EY : TE :: GD - DF : GD$, seu $DY : TE :: GD - DF : GD$, & permutando $DY : GD - DF :: TE : GD$; proindeque $2DY$ erit aequalis differentiae rectarum GD, & DF. Quamobrem $DT^2 = GD \times DY = 2DE \times DY = 2DY \times DE = DE \times (GD - DF)$.

§. IV.

Si describatur Parabola Apolloniana GX, cujus *Fig. 20.* vertex G, & parameter tertia proportionalis ad latus potentiae Hyperbolae TE, & dimidium axem transversum TD; transibit Parabola per punctum S, nempe intersectionis tangentis verticalis Hyperbolae, & assymptoti.

Ex

Ex puncto S agatur recta SV normalis rectae SG , quae occurrat asymptoto PD producto, & demittatur perpendiculum SA . Quoniam rectae SV , TD sunt parallelae inter se, erunt DG , DV inter se aequales; erit quoque recta DF aequalis rectae DA , quia triangula GDF , SDA sunt aequalia, & similia. Quare recta AV erit aequalis summae rectarum GD , DF ; proindeque $SA = GA \times AV = GA \times (GD + DF)$. At vero summa rectarum GD , DF est tertia proportionalis ad TE , & TD (§. I.), seu parameter Parabolae $G X$. Igitur Parabola $G X$ transit per punctum S .

§. V.

Si Hyperbola est aequilatera, Parabola transibit per punctum S , si parameter fuerit DG ; quandoquidem hoc in casu recta DF evanescit.

§. VI.

Fig. 21.

Sit modo angulus asymptotorum PDR obtusus; Parabola $G X$, cujus apex G , & parameter tertia proportionalis ad TE , & TD , transibit quoque per punctum S .

Rectae GD , DF aequantur rectis DV , DA , quia DT , & VS sunt parallelae inter se, & triangula DGF , DSA sunt aequalia, & similia. Quamobrem VA erit aequalis differentiae rectarum GD , ac DF ;

consequenter $SA = GA \times AV = GA \times (GD - DF)$. Sed $GD - DF$ est tertia proportionalis ad TE , & DT (§. III.). Ergo punctum S est in Parabola $G X$.

§. VII.

§. VII.

Quadratum semiaxis transversi DT Hyperbolae sca- *Fig. 20.*
lenae, five aequilaterae KTH aequatur dimidio qua- 21. 22.
drati semiordinatae Parabolae DC , seu DS .

Sit primo Hyperbola KTH scalena, cujus assym- *Fig. 20.*
ptota DR , DP angulum acutum comprehendant.

Quadratum $\overline{DC} = GD \times (GD + DF)$ (§. IV.), seu
 $\overline{DC} = \overline{GD} + GD \times DF$. Quoniam vero trigona TEY ,
 GDF sunt similia, erit $TE : EY :: GD : DF$, &
permutando $TE : GD :: EY : DF$; proindeque re-
cta EY erit dimidium rectae DF . Quapropter $\overline{DC} =$
 $4\overline{DE} + 2DE \times 2EY = 2\overline{DE} + 2\overline{ET} + 4DE \times EY =$
 $2\overline{DT}$; consequenter $\overline{DT} = \frac{1}{2}\overline{DC}$.

Quod si Hyperbola fuerit aequilatera, angulus GDS *Fig. 22.*
rectus erit, ideoque $\overline{DS} = \overline{DT} + \overline{TS}$. At vero DT
aequatur rectae TS , quia anguli SDT , TSD sunt
aequales inter se. Igitur $\overline{DS} = 2\overline{DT}$, & $\overline{DT} =$
 $\frac{1}{2}\overline{DS}$.

Sit denique angulus assymptotorum PDR obtusus. *Fig. 21.*

$\overline{DC} = GD \times (GD - DF)$ (§. VI.), idest $\overline{DC} =$
 $\overline{GD} - GD \times DF$. Deinde, quia triangula TEY ,
 GDF sunt similia, erit $TE : GD :: EY : DF$, con-
sequenter recta DF erit dupla rectae EY . Quare
 $\overline{DC} = \overline{GD} - GD \times DF = 4\overline{DE} - GD \times 2EY =$
 $2\overline{DE}$

$$2\overline{DE}^2 + 2\overline{ET}^2 - 4DE \times EY = 2\overline{DT}^2, \text{ proindeque } \overline{DT}^2 = \frac{1}{2}\overline{DC}^2.$$

§. VIII.

Fig. 22. Si latus ET potentiae Hyperbolae aequilaterae KTH , producat^{ur} usque ad Parabolam GNS ; erit applicata EN aequalis semiaxi DT , seu TG .

Etenim quadratum $\overline{EN}^2$ aequatur rectangulo ex EG in GD (§. V.). Atqui etiam quadratum $\overline{TG}^2$ aequatur rectangulo ex EG in GD . Ergo EN aequatur rectae TG , seu TD .

§. IX.

Fig. 23. Sit curva BTQ Hyperbola cubica, & aequilatera, cujus tangens verticalis STG , sitque NM quaelibet applicata; si accipiat^{ur} GK in asymptoto DP aequalis NM , atque ex puncto K elevetur perpendiculum KH aequale lateri TE , describaturque Parabola Apolloniana GF , cujus apex punctum G , quaeque transeat per punctum H : erit quadratum semiorinatae DF aequale spatio interminato $MNDABM$.

Quoniam spatium $MNDABM$ aequatur $2\overline{TE}^3$: NM , erit $NM: 2\overline{TE}^2:: \overline{TE}^2: MNDABM$. At vero ex proprietate Parabolae est $GK: GD:: KH: \overline{DF}^2$, idest $NM: 2\overline{TE}^2:: \overline{TE}^2: \overline{DF}^2$. Quare $\overline{TE}^2: MNDABM:: \overline{TE}^2: \overline{DF}^2$, consequenter quadratum $\overline{DF}^2$ erit aequale spatio Hyperbolico $MNDABM$.

§. X.

§. X.

Si abscissa DN fuerit aequalis duplae DG, spa- *Fig. 24.*
tium interminatum MNDABM erit aequale qua-
drato $\overline{DG}^2$.

Methodo praecedentis §. describatur Parabola GHS.

Quoniam Hyperbola BTM est cubica, erit $\overline{TE}^3 =$
 $\overline{DN} \times \overline{NM}^2$, seu, quia $\overline{DN} = 4\overline{TE}$, $\overline{TE}^3 = 4\overline{TE}$
 $\times \overline{NM}^2$, & $\overline{TE} = 4\overline{NM}^2$; ideoque $\overline{TE} = 2\overline{NM}$, &
 $\frac{1}{2}\overline{TE} = \overline{NM}$. At vero $\overline{KG} = \overline{NM}$, consequenter
 $\overline{KG} = \frac{1}{2}\overline{TE}$; hinc, cum sit $\overline{KG} : \overline{DG} :: \overline{KH} :$
 $\overline{DS}$, erit $\frac{1}{2}\overline{TE} : 2\overline{TE} :: \overline{TE} : \overline{DS} = 4\overline{TE}$.
Quamobrem $\overline{DS} = 2\overline{TE} = \overline{DG}$. Atqui spatium
 $\overline{MNDABM} = \overline{DS}^2$ (§. IX.). Ergo idem spatium
erit quoque aequale quadrato $\overline{DG}^2$.

§. XI.

Quapropter parameter Parabolae GHS erit recta
GD; & punctum K erit focus ejusdem Parabolae,
quia $\overline{KG} = \frac{1}{2}\overline{TE} = \frac{1}{4}\overline{DG}$, nempe quartae parti
parametri.

§. XII.

Tangens verticalis SG secat bifariam applicatam
HK, itidemque MO parallela affymptoto DN.

Quia RK est parallela lateri TE, erunt trigona
F TEG,

TEG , RKG similia, consequenter $TE:EG::RK:KG$. Sed $TE=EG$; ergo etiam $RK=KG$. At vero punctum K est focus Parabolae (§. XI.), ideoque KH est dupla KG , seu KR ; quare HK secta est bifariam in puncto R a tangente verticali STG .

Praeterea $MN=\frac{1}{2}TE$ (§. X.), ideoque etiam $KR=\frac{1}{2}TE$. Sed $TE=HK$; igitur $KR=\frac{1}{2}HK$.

§. XIII.

Quadratum applicatae Parabolae KH ad quadratum applicatae Hyperbolae KP eandem habet rationem, quam 3 ad 2; quadratum vero EQ est duplum alterius ET .

Nam $KH^2=KG \times GD$ (§. XI.), seu $KH^2=\frac{1}{4}GD \times GD=\frac{1}{4}GD^2$. Dein $TE^2=DK \times KP$, idest $EG^2=\frac{3}{2}EG \times KP$, & $KP=\frac{2}{3}EG$. Quapropter erit $KH^2:KP^2::\frac{1}{4}GD^2(=EG^2):\frac{2}{3}EG^2$, seu $KH^2:KP^2::3EG^2:2EG^2::3:2$.

Deinde $EQ^2=EG \times GD=2EG^2$, & $ET^2=EG^2$; quare $EQ^2:ET^2::2EG^2:EG^2::2:1$; ideoque $EQ^2=2ET^2$.

§. XIV.

Fig. 25. Sit modo etiam Parabola GS secundi generis, cuius parameter GD , sitque TC tertia pars lateris TE ,

43

TE, & CM parallela affymptoto DN; spatium Hyperbolicum interminatum MNDABM erit aequale spatio Parabolico-cubico DSG.

Etenim spatium DSG $\equiv \frac{3}{4} DS \times DG \equiv \frac{3}{4} \overline{DG}^2 \equiv \frac{3}{4} \times 4 \overline{EG}^2 \equiv 3 \overline{EG}^2$. Spatium vero Hyperbolicum MNDABM $\equiv 2 \overline{TE}^3 : MN \equiv 2 \overline{EG}^3 : MN$. At vero TC $\equiv \frac{1}{3} TE$, ideoque CE $\equiv \frac{2}{3} TE \equiv \frac{2}{3} EG \equiv MN$. Quare spatium MNDABM $\equiv 2 \overline{EG}^3 : MN \equiv 2 \overline{EG}^3 : \frac{2}{3} EG \equiv 3 \overline{EG}^2$; consequenter spatium DSG aequatur spatio MNDABM.



TE, & CM paralleli asymptoto DM; punctum H
per punctum intermedium MNDAM in asymptoto
spatio Parabolo-cubico DSC.

Etiam punctum DSG = DSNDG = DDC =

3EG = 3EG. Spatium vero Hyperbolicum

MNDAM = TE: MN = 2EG: MN. At vo

to TC = TR, ideoque CE = TE = EG =

MN. Quare spatium MNDAM = 4EG: MN =

2EG: 1/2EG = 3EG: correspondens spatium DSG

ad punctum MNDAM.

Ad punctum MNDAM, & ad punctum

TE, & CM paralleli asymptoto DM; punctum H

per punctum intermedium MNDAM in asymptoto

spatio Parabolo-cubico DSC.

Etiam punctum DSG = DSNDG = DDC =

3EG = 3EG. Spatium vero Hyperbolicum

MNDAM = TE: MN = 2EG: MN. At vo

to TC = TR, ideoque CE = TE = EG =

MN. Quare spatium MNDAM = 4EG: MN =

2EG: 1/2EG = 3EG: correspondens spatium DSG

ad punctum MNDAM.

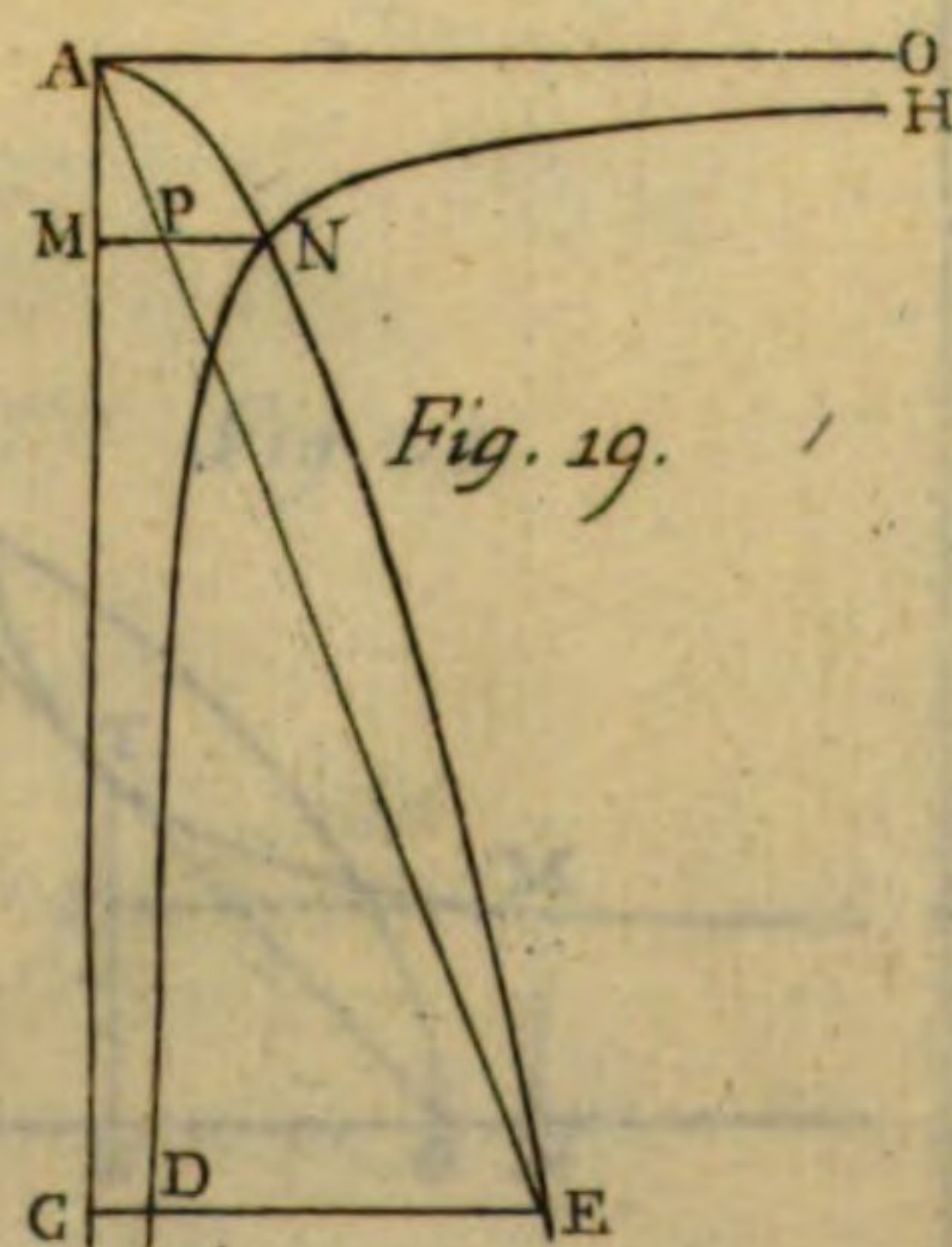
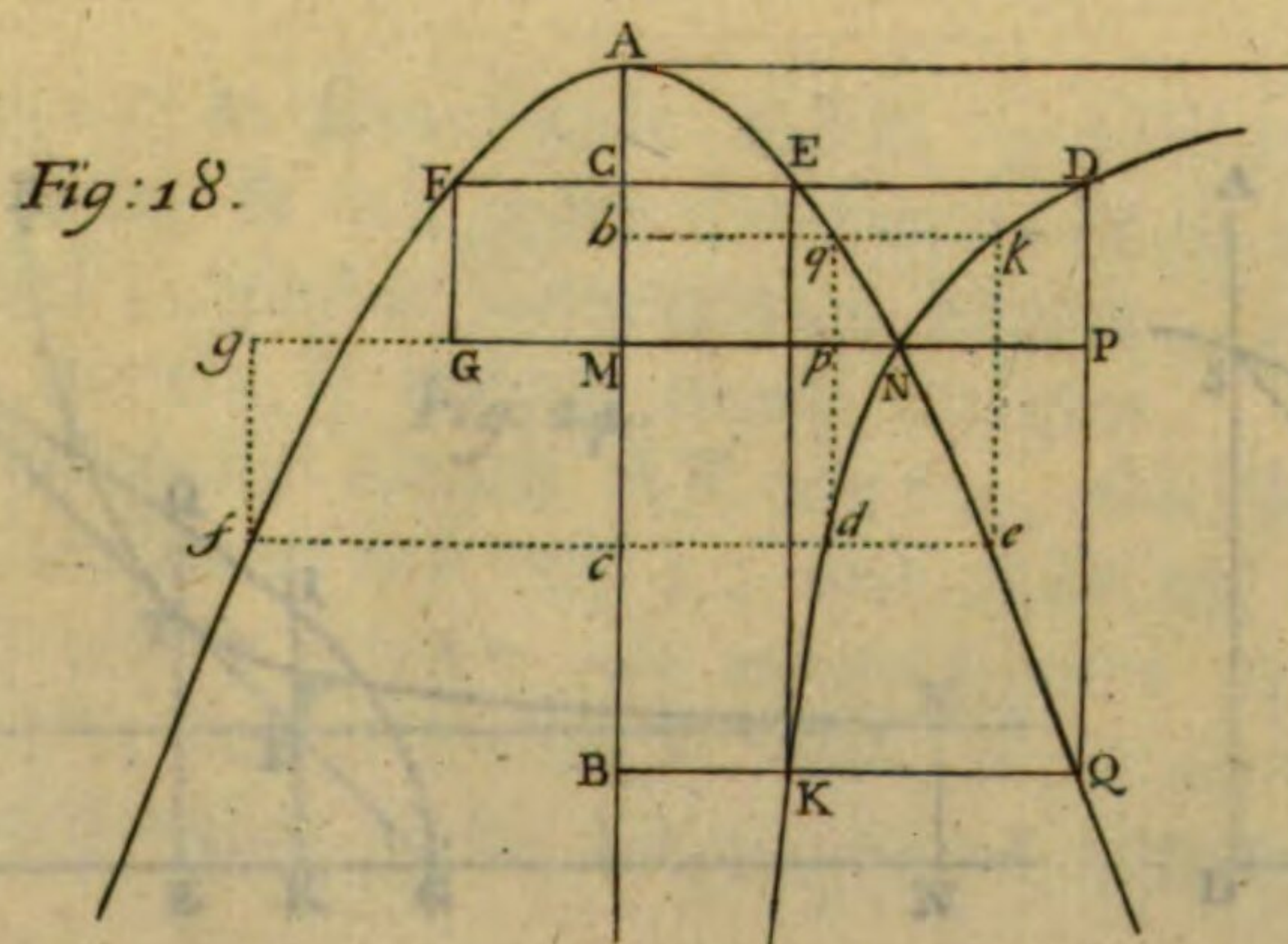
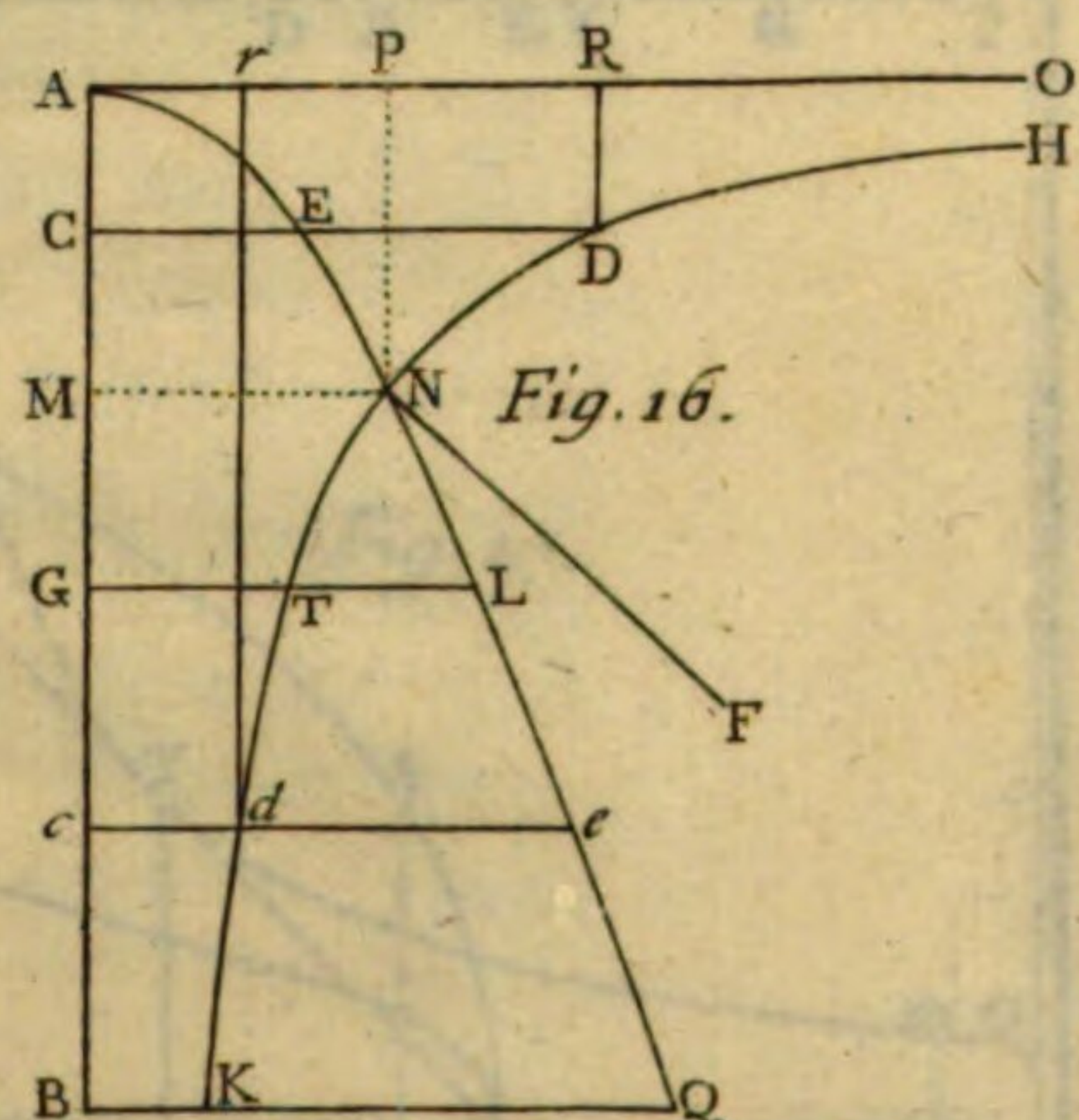
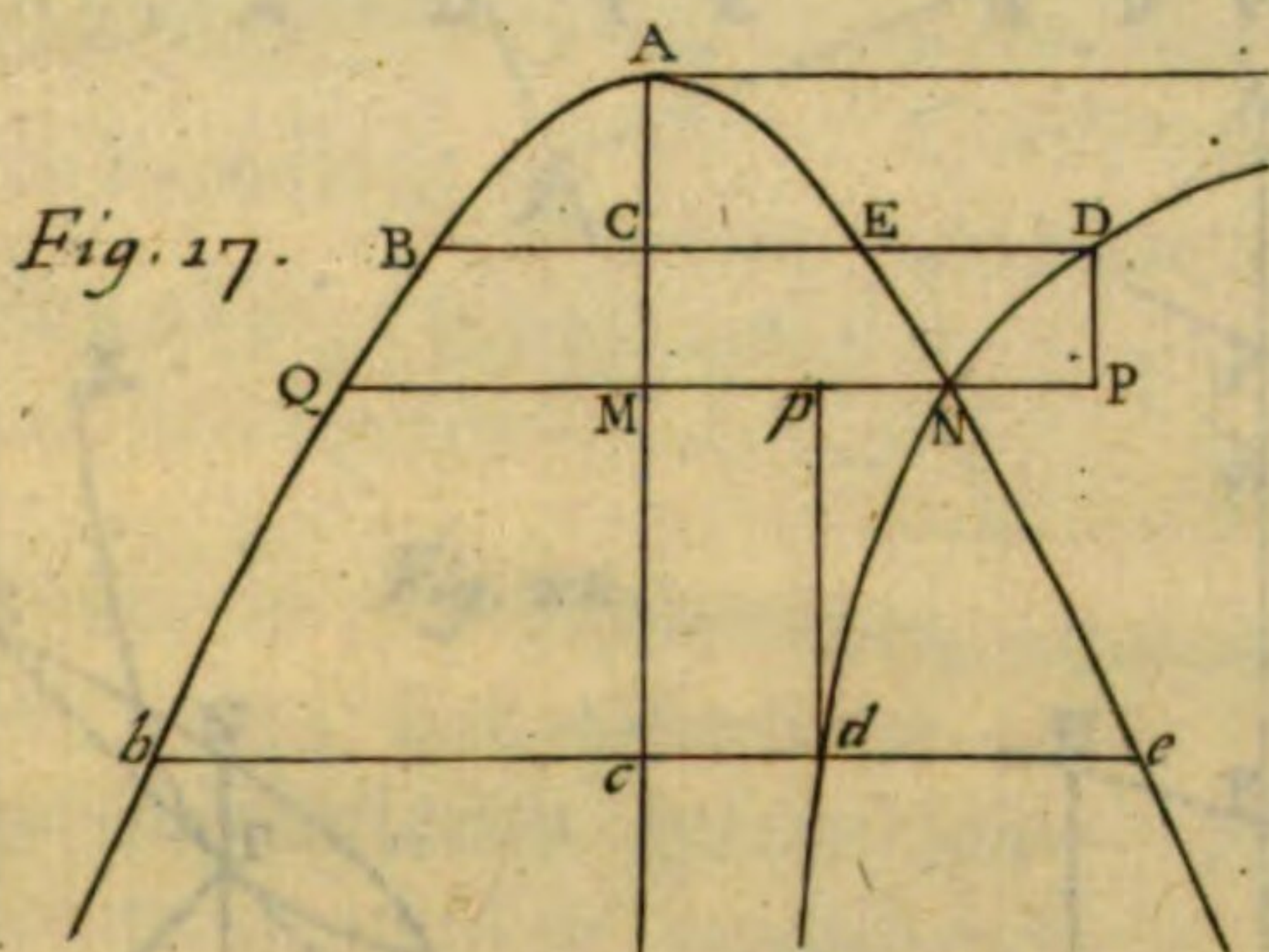
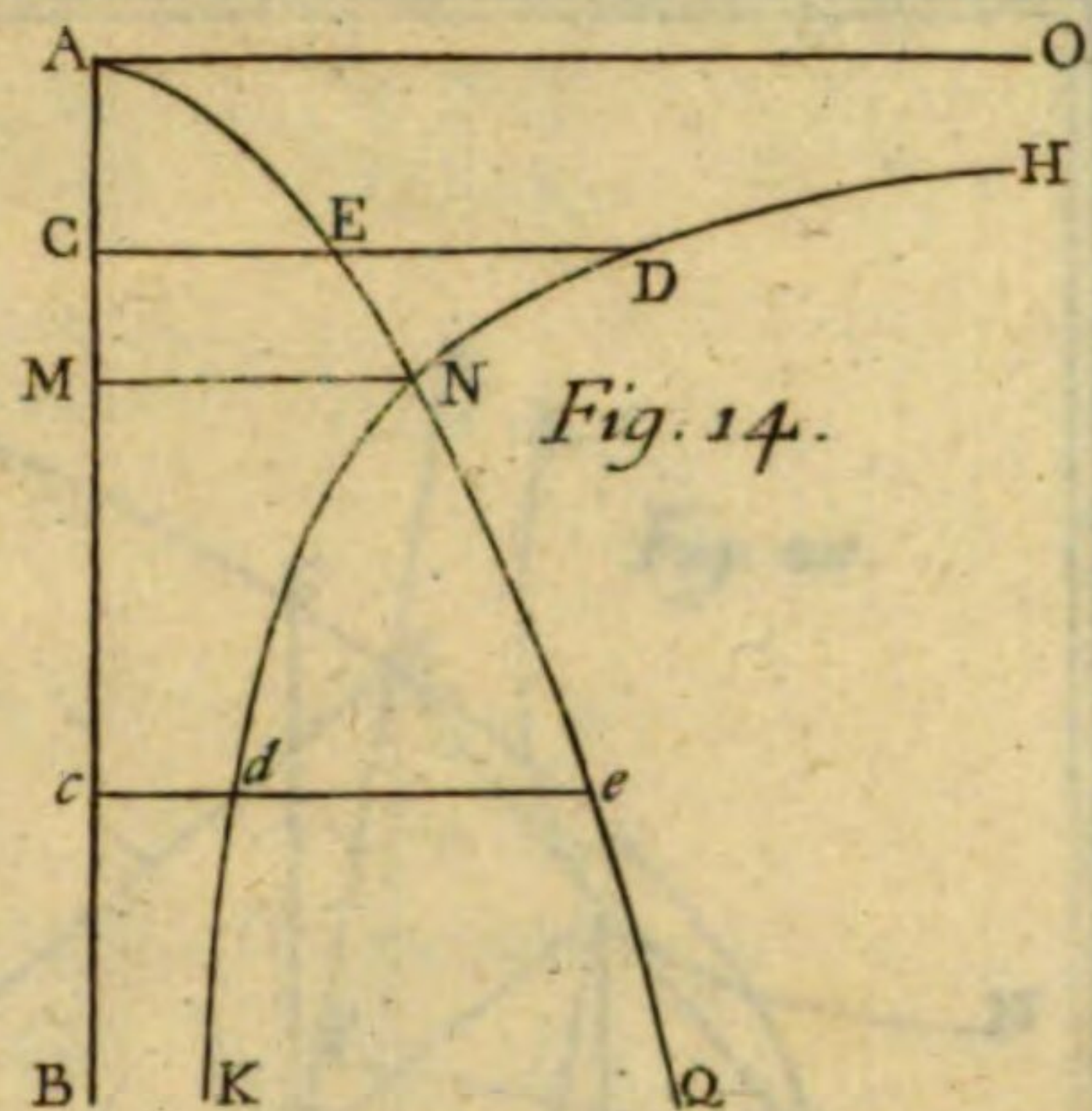
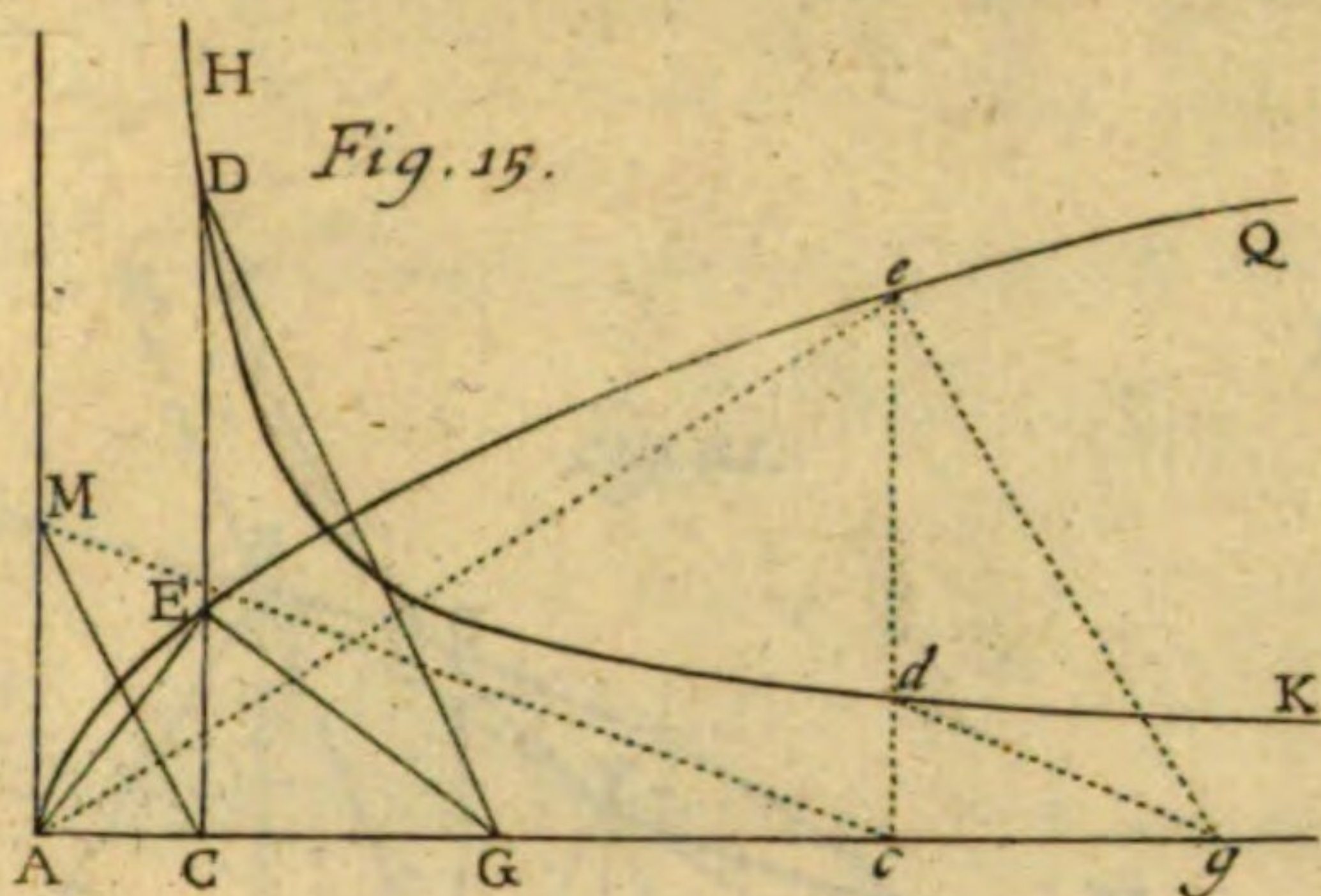
Ad punctum MNDAM, & ad punctum

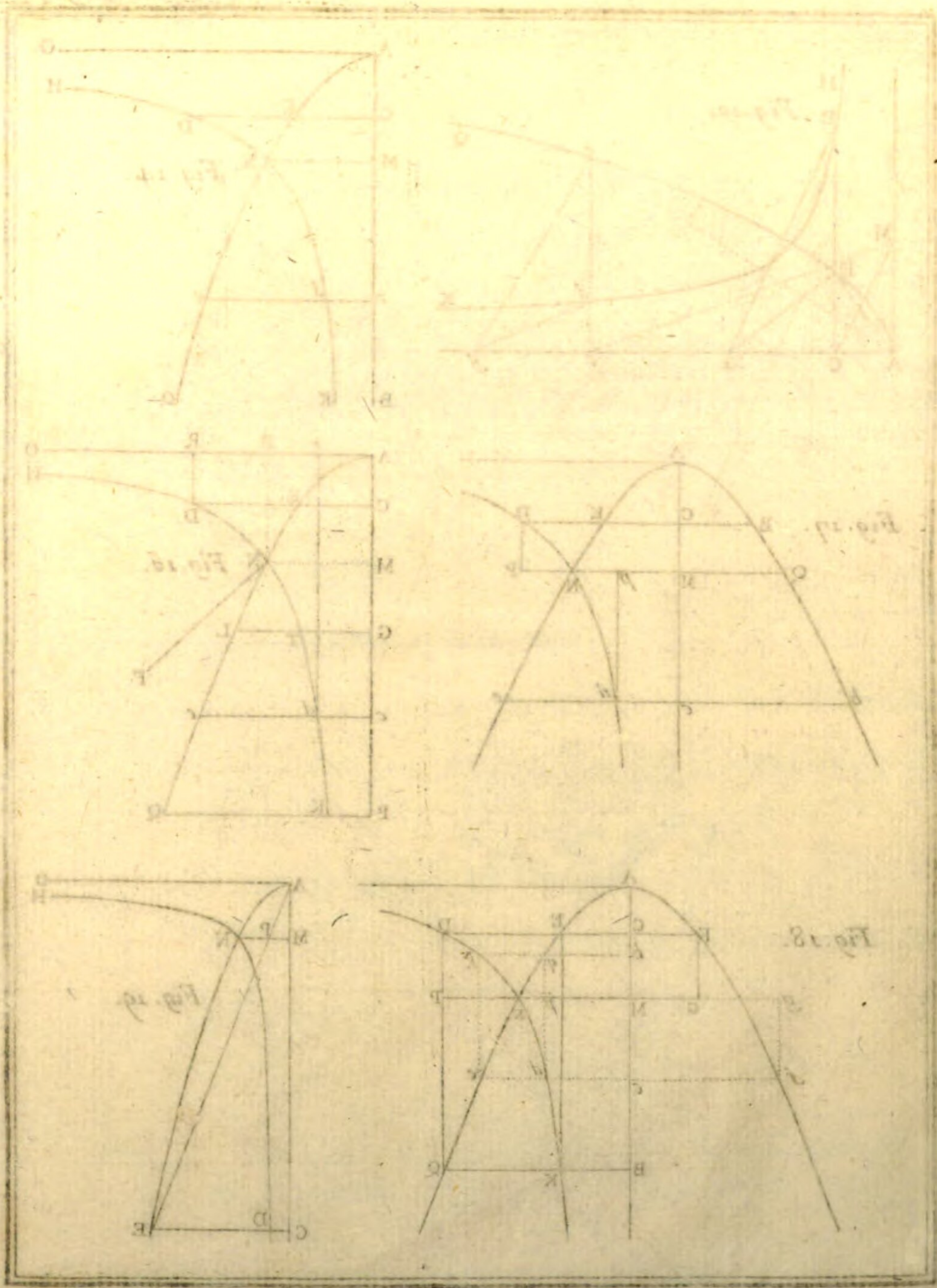
TE, & CM paralleli asymptoto DM; punctum H

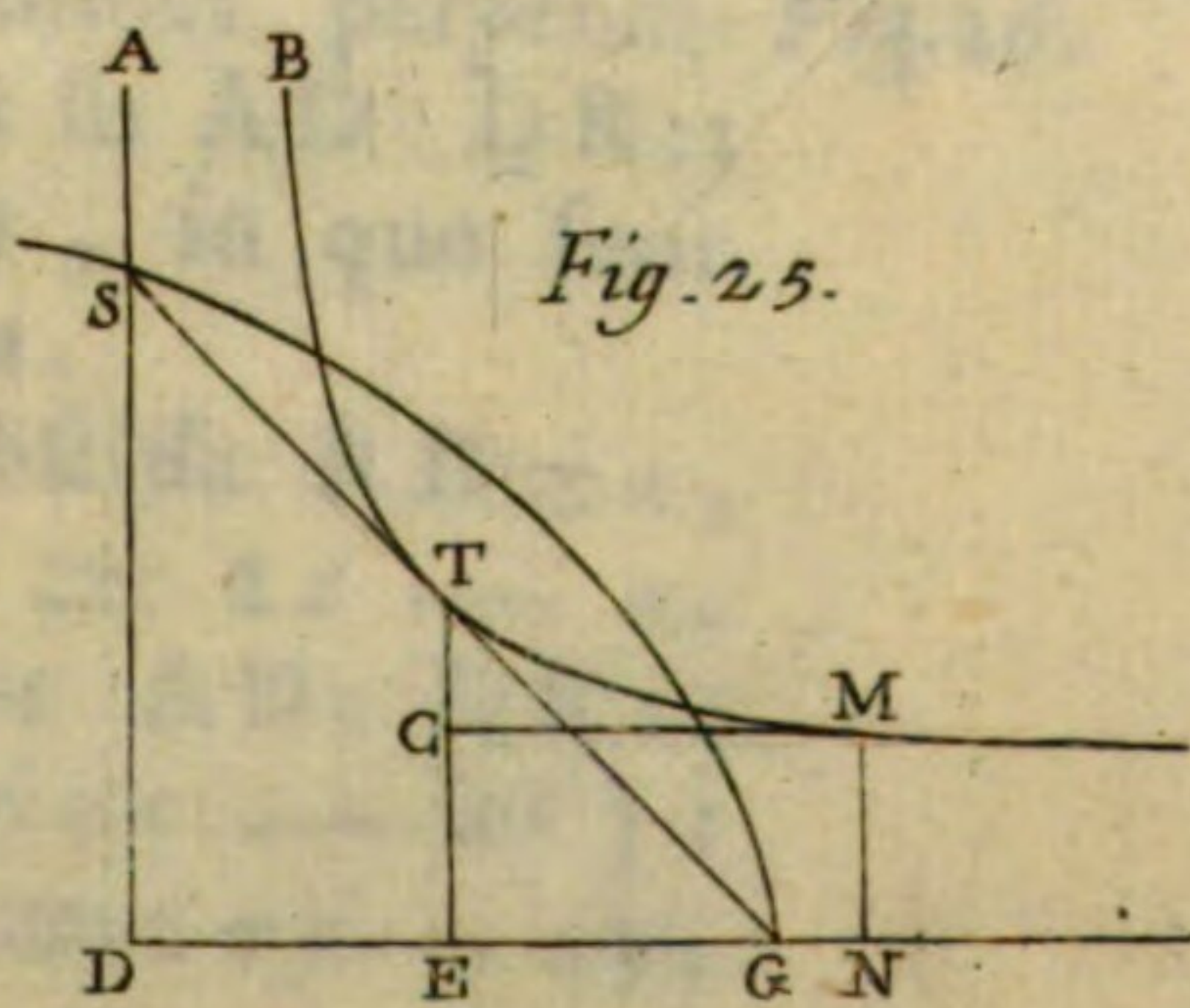
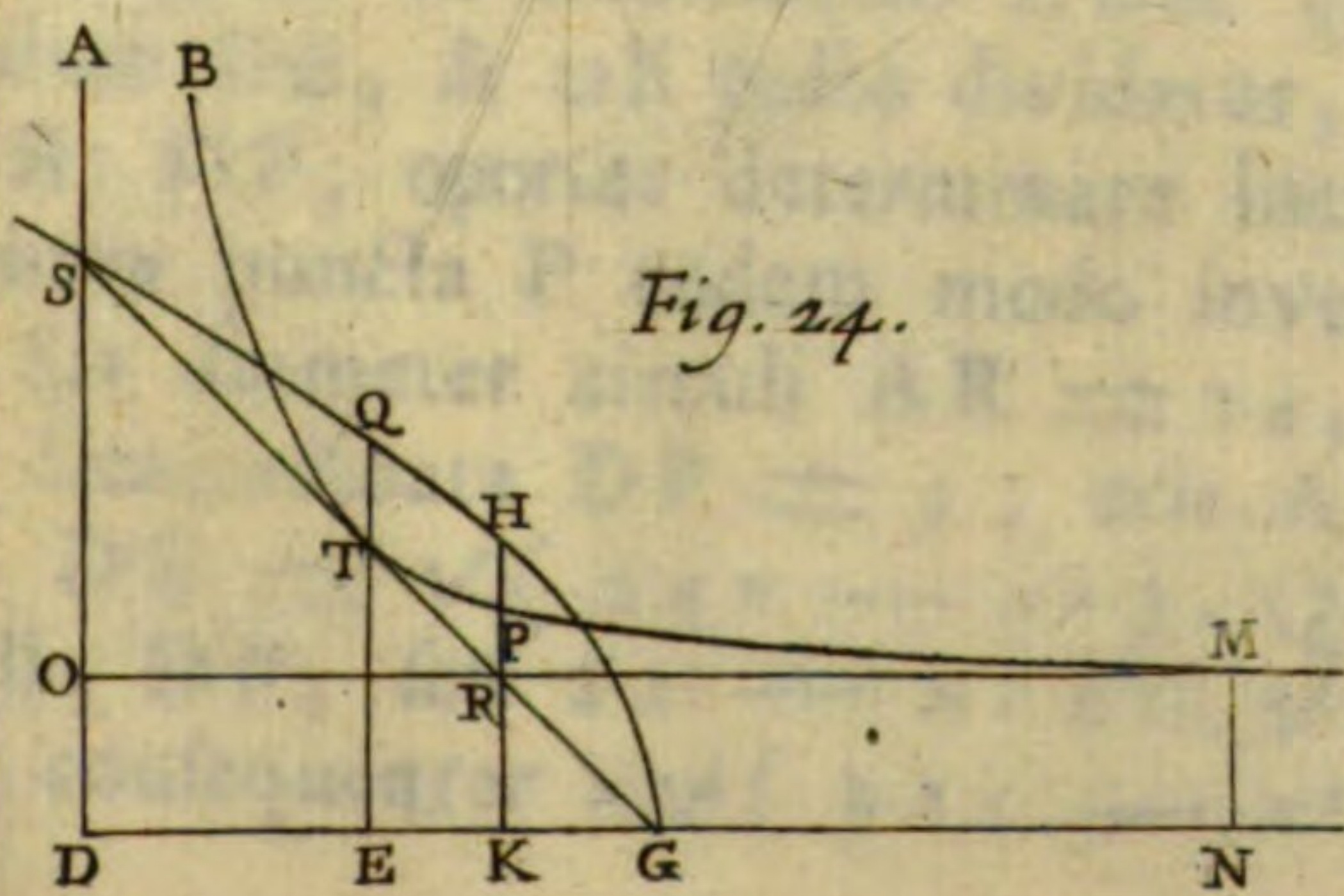
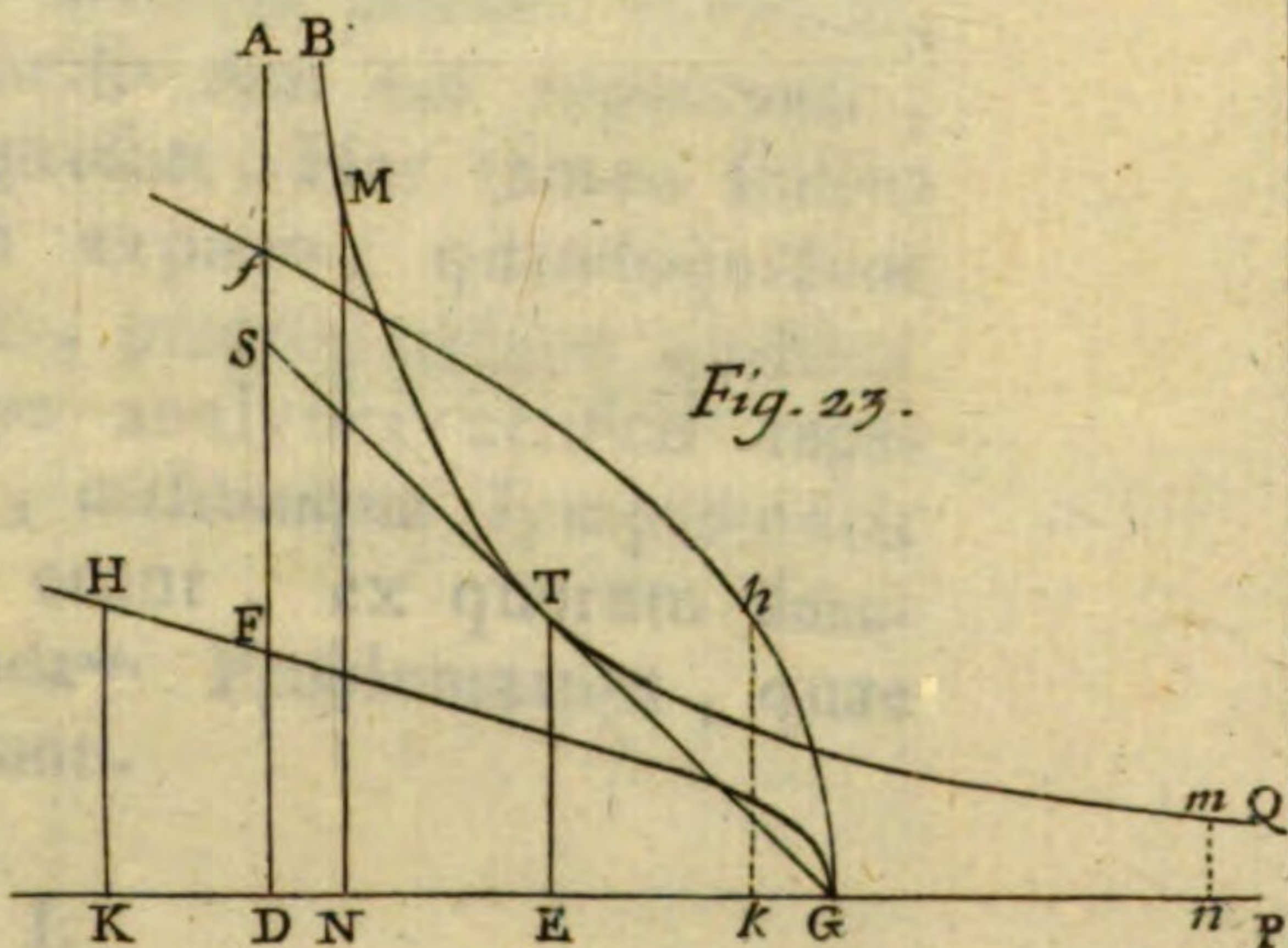
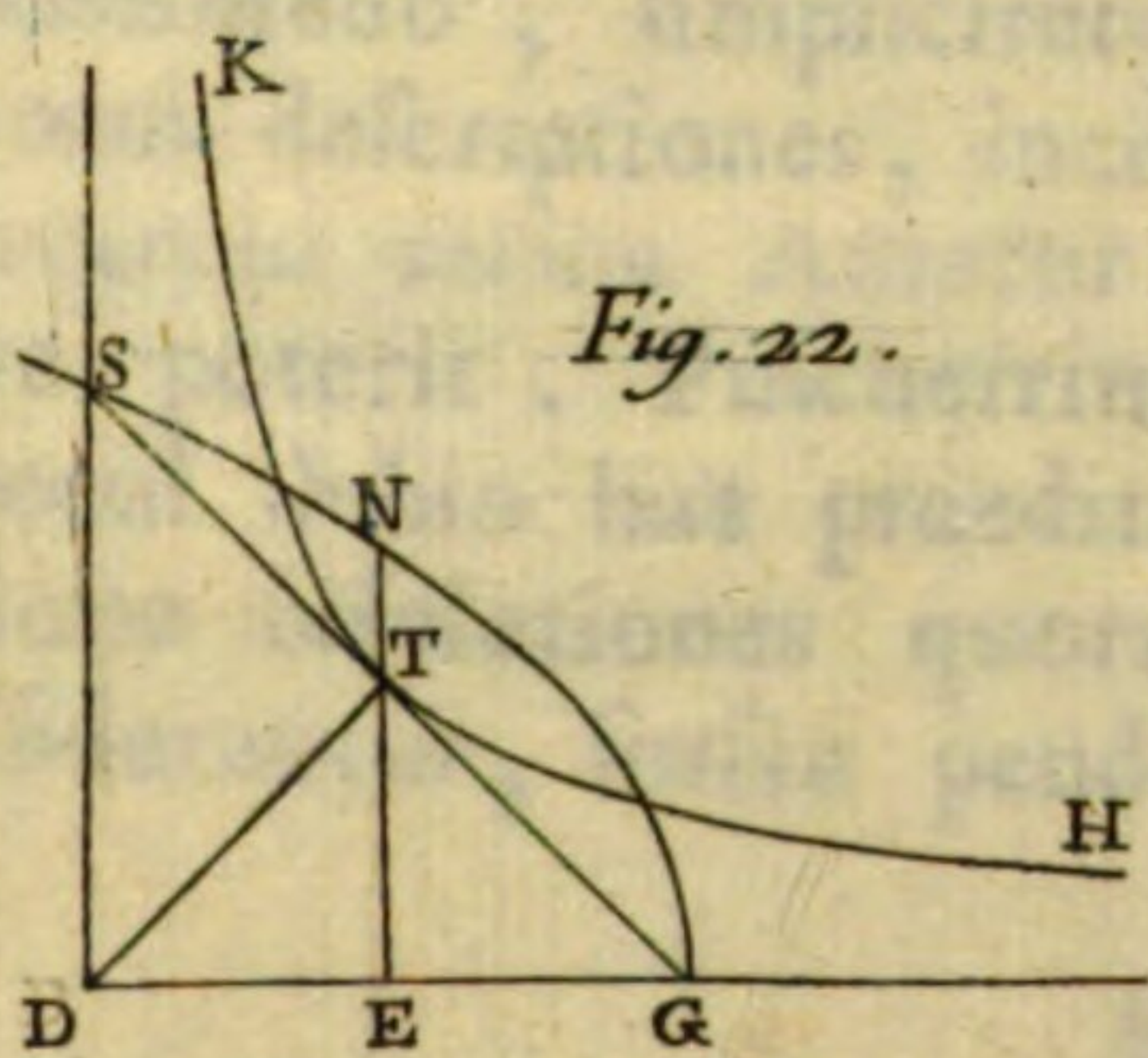
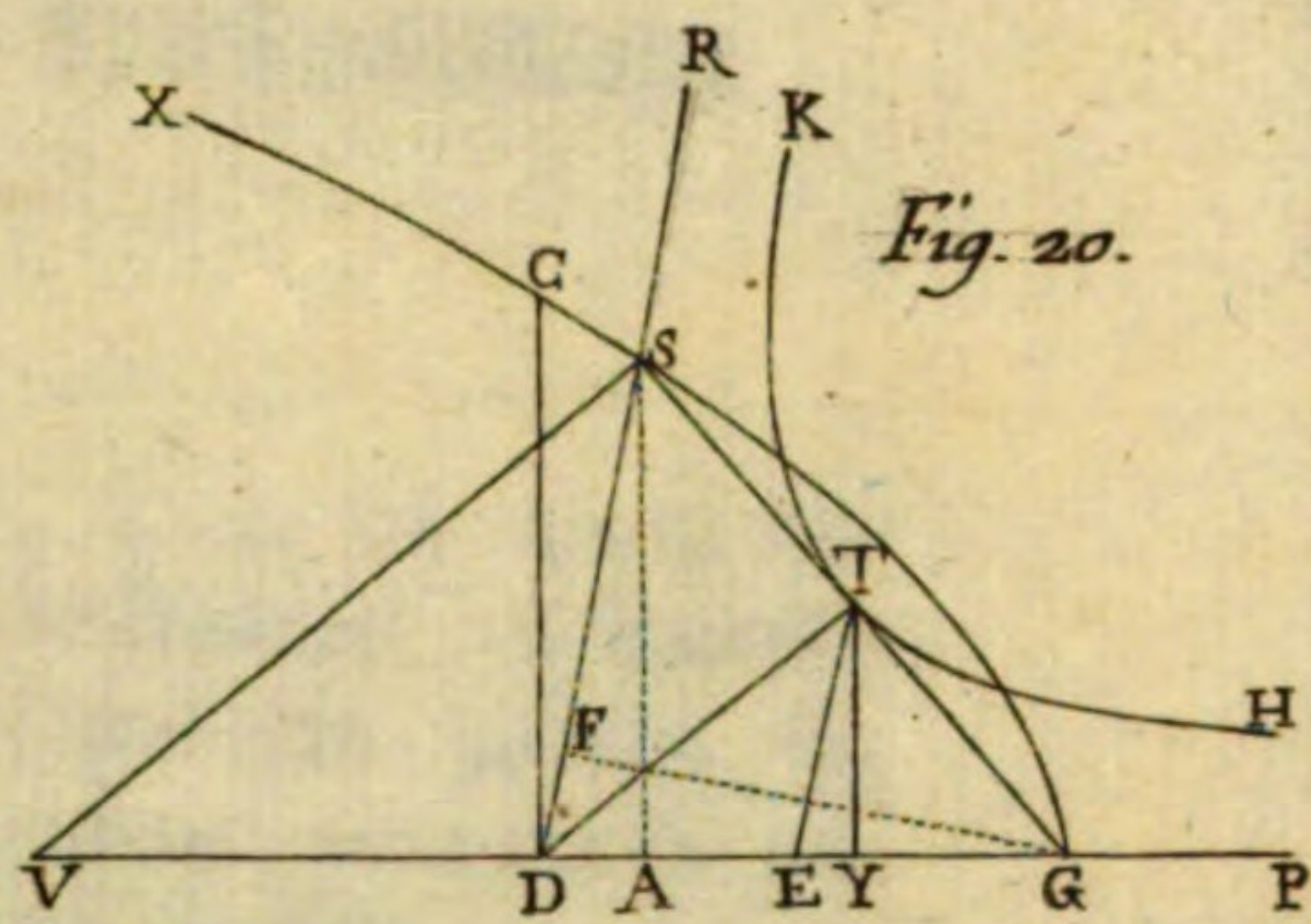
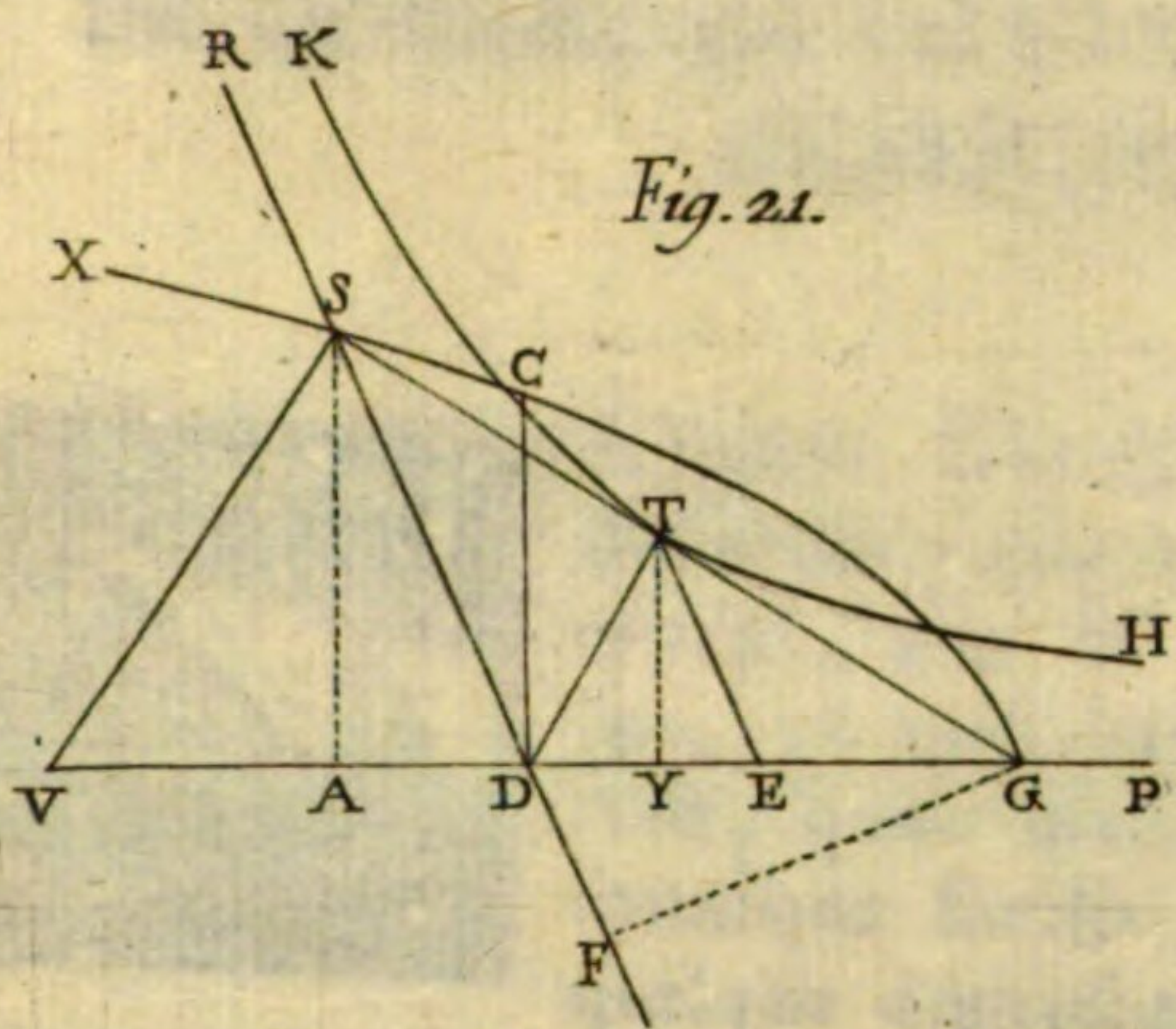
per punctum intermedium MNDAM in asymptoto

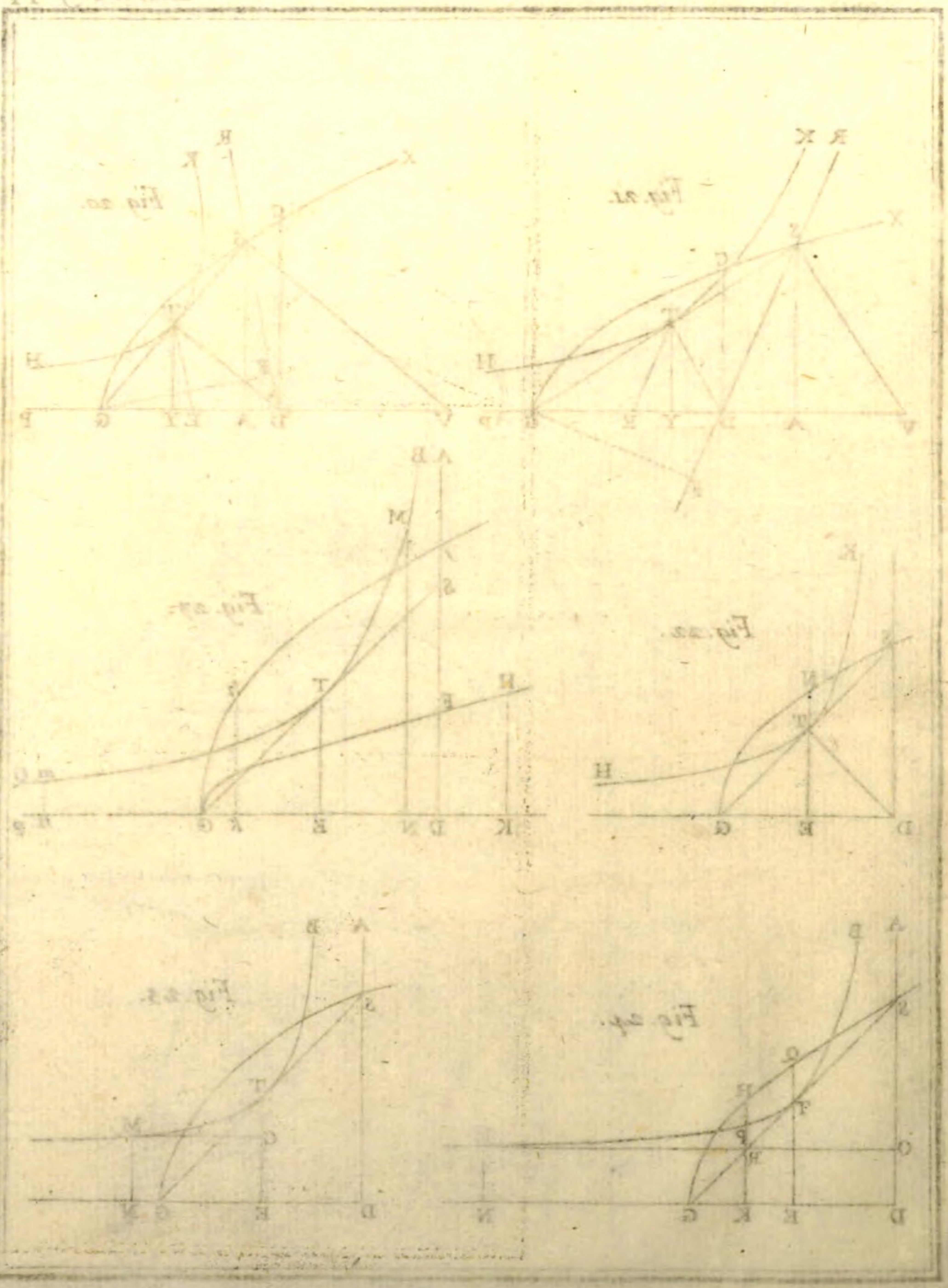
spatio Parabolo-cubico DSC.

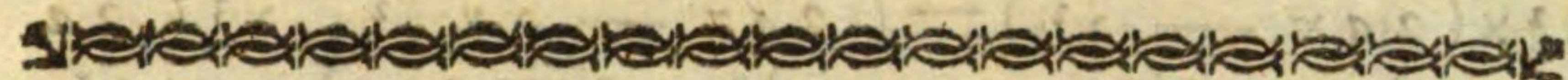
Etiam punctum DSG = DSNDG = DDC =











OPUSCULUM III.

DE QUIBUSDAM CURVIS SUPERIORUM GENERUM.



Nonnullas Curvas, quas ex varia semicirculi, atque rectarum collatione inveni, hoc in Opusculo palam facio, ex quo ingentem numerum earum, quae ulterius possent inveniri, simulque facilitatem eas reperiendi, quisque dignoscet. Has tamen indico solummodo, simpliciterque expono; quandoquidem earum descriptiones, incessus, proprietatesque quilibet novarum rerum Amator ope analytici artificii reperire poterit. Pulcherrimis, utilibusque symptomatis procul dubio hae praeditae erunt, ex quorum detectione resolutiones quorundam Problematum, quae desiderantur, forsan pendeant.

§. I.

Erigatur in semicirculo ABR quodlibet perpendiculum DB , & tali pacto dividatur, ut sit $AD:DR::DB:DP$; oportet determinare locum, in quo sunt omnia puncta P eodem modo inventa. Fig. 26.

Sit diameter circuli $AR = 2a$, abscissa $RD = x$, & semiordinata $DP = y$; erit $AD = 2a - x$, & $DB = \sqrt{2ax - x^2}$. Quare $AD:DR::DB:DP$, seu $2a - x : x :: \sqrt{2ax - x^2} : y$; consequenter $x\sqrt{2ax - x^2} = 2ay - xy$,
 $x^2 x$

$x^2 \times (2ax - x^2) = (2ay - xy)^2$, $x^3 (2a - x) = y^2 (2a - x)^2$, $x^3 = 2ay^2 - xy^2$,
 $\& x^3 + y^2x - 2ay^2 = 0$. Quapropter locus pun-
 ctorum P est Curva secundi generis.

§. II.

Si normalis DB extrema, & media ratione fe-
 cetur in P, ita ut sit $BD: DP:: DP: PB$; locus
 punctorum P erit Curva tertii generis.

Etenim erit $PB = \sqrt{(2ax - x^2)} - y$; con-
 sequenter, quia $PD^2 = DB \times BP$, $y^2 = \sqrt{(2ax - x^2)} (\sqrt{(2ax - x^2)} - y) = 2ax - x^2 - y\sqrt{(2ax - x^2)}$,
 $y^2 - 2ax + x^2 = -y\sqrt{(2ax - x^2)}$, $y^4 - 4axy^2 + 4a^2x^2 + 2y^2x^2 - 4ax^3 + x^4 = 2axy^2 - x^2y^2$,
 $x^4 - 4ax^3 + 3y^2x^2 + 4a^2x^2 - 6ay^2x + y^4 = 0$, quae ad Curvam ter-
 tii generis pertinet.

Quod si perpendicularum DB tali pacto dividatur in
 P, ita ut sit $DB: BP:: BP: PD$; erit quoque lo-
 cus punctorum P Curva tertii ordinis. Nam $BP^2 = BD \times DP$, seu $(\sqrt{(2ax - x^2)} - y)^2 = y\sqrt{(2ax - x^2)}$,
 $2ax - x^2 - 2y\sqrt{(2ax - x^2)} + y^2 = y\sqrt{(2ax - x^2)}$, $2ax - x^2 + y^2 = 3y\sqrt{(2ax - x^2)}$,
 $\& 4a^2x^2 + x^4 + y^4 - 4ax^3 - 2x^2y^2 + 4axy^2 = 18axy^2 - 9y^2x^2$, idest $x^4 - 4ax^3 + 7y^2x^2 + 4a^2x^2 - 14ay^2x + y^4 = 0$.

§. III.

Agatur modo in semicirculo quaelibet chorda AB,
 jungaturque BR, atque ex puncto B demittatur per-
 pendiculum BD, fiatque DP quarta proportionalis
 ad

ad AB, RB, & RC; oportet invenire locum pun-
ctorum P.

Quia $AD = 2a - x$, & $DB = \sqrt{2ax - x^2}$;
erit $AB = \sqrt{4a^2 - 4ax + x^2 + 2ax - x^2} =$
 $\sqrt{4a^2 - 2ax}$, & $BR = \sqrt{2ax}$. Quare
 $\sqrt{4a^2 - 2ax} : \sqrt{2ax} :: a : y$; & $a\sqrt{2ax} =$
 $y\sqrt{4a^2 - 2ax}$, $2a^3x = 4a^2y^2 - 2axy^2$,
 $a^2x = 2ay^2 - xy^2$, $y^2x + a^2x - 2ay^2 = 0$,
quae Curvam secundi ordinis definit.

Sit DG tertia continue proportionalis ad chordam
BR, & radium CR. Erit igitur $\overline{CR} = BR \times DG$,
feu $a^2 = y\sqrt{2ax}$, & $a^4 = 2axy^2$, $2xy^2 -$
 $a^3 = 0$, quae ad Curvam pariter secundi ordinis
pertinet.

§. IV.

Producatur diameter AR in K, ita ut sit RK *Fig.27.*
aequalis radio CR, & ex puncto K agatur recta
KB, quae occurrat cuilibet circumferentiae puncto
B, deinde accipiatur DP quarta proportionalis ad
BK, KR, & RD; locus punctorum P erit Curva
secundi generis.

Quoniam $RK = RC$, erit $DK = x + a$; ideo-
que $BK = BD + DK = 2ax - x^2 + x^2 +$
 $2ax + a^2 = 4ax + a^2$, & $BK = \sqrt{4ax + a^2}$.
Quare erit $BK : KR :: RD : DP$, feu $\sqrt{4ax + a^2} :$
 $a :: x : y$; consequenter $ax = y\sqrt{4ax + a^2}$,
 $a^2x^2 = 4axy^2 + a^2y^2$, $ax^2 - 4y^2x - ay^2$
 $= 0$.

Sit DP quarta proportionalis post KR, KB, &
RD. Erit $a : \sqrt{4ax + a^2} :: x : y$, & $x\sqrt{4ax$
 $+ a^2} = ay$, $a^2y^2 = 4ax^3 + a^2x^2$, $4x^3 + ax^2$

— $ay^2 = 0$, quae ad Curvam secundi ordinis similiter spectat.

Fiat $BK : KD :: DR : DP$; erit $\sqrt{4ax + a^2} : a + x :: x : y$; ideoque $ax + x^2 = y\sqrt{4ax + a^2}$, $a^2x^2 + 2ax^3 + x^4 = 4axy^2 + a^2y^2$, & $x^4 + 2ax^3 + a^2x^2 - 4ay^2x - a^2y^2 = 0$. Quare locus punctorum P erit Curva tertii generis.

Si vero fiat $KD : KB :: DR : DP$; erit $a + x : \sqrt{4ax + a^2} :: x : y$, consequenter $ay + xy = x\sqrt{4ax + a^2}$, & $a^2y^2 + 2axy^2 + x^2y^2 = 4ax^3 + a^2x^2$, $4ax^3 - y^2x^2 + a^2x^2 - 2ay^2x - a^2y^2 = 0$, quae ad Curvam pariter tertii ordinis pertinet.

Sit denique DK media proportionalis inter BK, & DG; emerget $a^2 + 2ax + x^2 = y\sqrt{4ax + a^2}$, & $a^4 + 4a^3x + 4a^2x^2 + 2a^2x^2 + 4ax^3 + x^4 = 4axy^2 + a^2y^2$, $x^4 + 4ax^3 + 6a^2x^2 - 4ay^2x + 4a^3x - a^2y^2 + a^4 = 0$. Igitur Curva punctorum G est tertii generis.

§. V.

Ex puncto R erigatur normalis RH, & fiat $KR : RH :: RH : DP$; locus punctorum P erit Curva secundi ordinis.

Etenim $KD : DB :: KR : RH$, nempe $a + x : \sqrt{2ax - x^2} :: a : \frac{a\sqrt{2ax - x^2}}{a + x} = RH$.

Quapropter $a : \frac{a\sqrt{2ax - x^2}}{a + x} :: \frac{a\sqrt{2ax - x^2}}{a + x} : y$;

ideoque $ay = \frac{2a^3x - a^2x^2}{(a + x)^2}$, & $ay(a + x)^2 = 2a^3x - a^2x^2$, $yx^2 + ax^2 + 2ayx - 2a^2x + a^2y = 0$.

Sit

Sit $DR:RH::RH:DG$, seu $x:\frac{a\sqrt{(2ax-x^2)}}{a+x}::$

$$\frac{a\sqrt{(2ax-x^2)}}{a+x}:y; \text{ erit } yx = \frac{2a^3x - a^2x^2}{a^2 + 2ax + x^2},$$

& $a^2yx + 2axyx^2 + yx^3 = 2a^3x - a^2x^2$, $yx^2 + 2axyx + a^2x + a^2y - 2a^3 = 0$, quae spectat ad Curvam secundi generis.

Fiat KH media proportionalis inter DK , & DG . Quoniam $KD:KB::KR:KH$, erit $a+x:\sqrt{(4ax+a^2)}::a:\frac{a\sqrt{(4ax+a^2)}}{a+x} = KH$. Quare

$$\frac{4a^3x + a^4}{a^2 + 2ax + x^2} = ay + yx, 4a^3x + a^4 = a^3y + 2a^2yx + ayx^2 + a^2yx + 2axyx^2 + yx^3 = a^3y + 3a^2yx + 3axyx^2 + yx^3, \& yx^3 + 3axyx^2 + 3a^2yx - 4a^3x + a^3y - a^4 = 0, \text{ per quam Curva tertii ordinis definitur.}$$

Sit tandem DP aequalis perpendiculo RH . Erit itaque $\frac{a\sqrt{(2ax-x^2)}}{a+x} = y$, $ay + xy = a\sqrt{(2ax-x^2)}$, & $a^2y^2 + 2ay^2x + y^2x^2 = 2a^3x - a^2x^2$, $y^2x^2 + a^2x^2 + 2ay^2x - 2a^3x + a^2y^2 = 0$. Quare puncta P dant locum Curvae tertii generis.

§. VI.

Sit DE perpendicularis rectae BK , & DP aequalis eidem perpendiculo DE ; puncta P erunt locus Curvae tertii generis.

Quoniam trigona DEK , DKB sunt similia, erit $KB:BD::KD:DE$, idest $\sqrt{(4ax+a^2)}:\sqrt{(2ax-x^2)}::$
G
—

$$-x^2) :: a+x : \frac{(a+x)\sqrt{(2ax-x^2)}}{\sqrt{(4ax+a^2)}}. \text{ Quamo-}$$

$$\text{brem } y = \frac{(a+x)\sqrt{(2ax-x^2)}}{\sqrt{(4ax+a^2)}}, y\sqrt{(4ax+a^2)}$$

$$= (a+x)\sqrt{(2ax-x^2)}, \& 4ay^2x + a^2y^2 = \\ (a^2 + 2ax + x^2)(2ax-x^2) = 2a^3x - \\ a^2x^2 + 4a^2x^2 - 2ax^3 + 2ax^3 - x^4 = 2a^3x \\ + 3a^2x^2 - x^4, x^4 - 3a^2x^2 + 4ay^2x - 2a^3x \\ + a^2y^2 = 0.$$

§. VII.

Caeterum advertendum, quod si producat^r KB usque in Q, ea omnia, quae diximus relate ad punctum B, eidem quoque puncto Q convenire; atque hac ratione omnia puncta, quae ad Curvas pertinent, eadem constructione invenientur.

§. VIII.

Fig. 28. Ex diametri extremitate R erigatur indefinitum perpendiculum RK, agaturque quaelibet recta AK, & fiat AB: BK :: KR: DL; oportet determinare locum punctorum L.

Quia triangula ABD, AKR sunt similia, erit AB: BK :: AD: DR, ideoque AD: DR :: KR: DL, & DR: AD :: DL: KR, seu $x: 2a-x ::$

$$y: \frac{2ay - xy}{x} = KR. \text{ Quare } AD: DB :: AR:$$

$$RK, \text{ nempe } 2a-x: \sqrt{(2ax-x^2)} :: 2a: \frac{2ay-xy}{x}; \text{ consequenter } 2a\sqrt{(2ax-x^2)} =$$

$$\frac{(2a-x)(2ay-xy)}{x}, 2ax\sqrt{(2ax-x^2)} =$$

$$\begin{aligned}
&= (2a - x)(2ay - xy), 4a^2x^3(2a - x)^5 \\
&= y^2(2a - x)^4, 4a^2x^3 = y^2(2a - x)^3 \\
&= 8a^3y^2 - 12a^2y^2x + 6ay^2x^2 - y^2x^3, \& \\
&y^2x^3 + 4a^2x^3 - 6ay^2x^2 + 12a^2y^2x - 8a^3y^2 \\
&= 0, \text{ quae ad Curvam quarti generis pertinet.}
\end{aligned}$$

Sit modo $AK:KR::DR:DP$: Quoniam $AD:AR::DB:RK$, seu $2a-x:2a::\sqrt{(2ax-x^2)}:$
 $\frac{2a\sqrt{(2ax-x^2)}}{2a-x}$, erit $RK = \frac{2a\sqrt{(2ax-x^2)}}{2a-x}$,

$$\& AK = \sqrt{4a^2 + \frac{4a^2(2ax-x^2)}{(2a-x)^2}}. \text{ Igi-}$$

$$\text{tur } \sqrt{4a^2 + \frac{4a^2(2ax-x^2)}{(2a-x)^2}} : \frac{2a\sqrt{(2ax-x^2)}}{2a-x}$$

$$:: x:y, \& \frac{2ax\sqrt{(2ax-x^2)}}{2a-x} = y$$

$$\times \sqrt{4a^2 + \frac{4a^2(2ax-x^2)}{(2a-x)^2}},$$

$$\frac{4a^2x^2(2ax-x^2)}{(2a-x)^2} = y^2 \left(4a^2 + \frac{4a^2(2ax-x^2)}{(2a-x)^2} \right),$$

$$\frac{4a^2x^3}{2a-x} = y^2 \left(4a^2 + \frac{4a^2x}{2a-x} \right), 4a^2x^3 =$$

$$y^2(8a^3 - 4a^2x + 4a^2x), 4a^2x^3 = 8a^3y^2, \\ x^3 - 2ay^2 = 0; \text{ proindeque Curva punctorum P} \\ \text{est secundi ordinis.}$$

§. IX.

Ex centro C agatur CH parallela rectae AK, fiat-
 que $AB:BK::CE:FG$; puncta G erunt in Cur-
 va secundi gradus.

Quoniam triangula CEF, AKR sunt similia, erit
 $CE:FE::AR:RK$, nempe $a-x:\sqrt{(2ax-x^2)}::$
 $G \quad 2 \quad 2a:$

$$2a : \frac{2a\sqrt{(2ax - x^2)}}{a - x} = RK. \text{ Similiter } CF:$$

$$\begin{aligned} CE :: AR : AK, \text{ nempe } a - x : a :: 2a : \frac{2a^2}{a - x} \\ = AK. \text{ Jam vero } AK : KR :: KR : KB, \text{ seu} \\ \frac{2a^2}{a - x} : \frac{2a\sqrt{(2ax - x^2)}}{a - x} :: \frac{2a\sqrt{(2ax - x^2)}}{a - x} : \\ \frac{4a^2(2ax - x^2)}{2a^2(a - x)} = \frac{4ax - 2x^2}{a - x}; \text{ ideoque } KB = \\ \frac{4ax - 2x^2}{a - x}. \text{ Quapropter cum sit } AB : BK :: CE : \\ FG, \text{ \& componendo } AK : BK :: CE + FG : FG; \\ \text{erit quoque } \frac{2a^2}{a - x} : \frac{4ax - 2x^2}{a - x} :: a + y : y, \text{ \&} \\ 2a^2y = 4a^2x - 2ax^2 + 4ayx - 2yx^2, yx^2 \\ + ax^2 - 2ayx - 2a^2x + a^2y = 0. \end{aligned}$$

§. X.

Sit BQ parallela diametro AR, & AB : BQ :: KQ : DP; puncta P eodem modo inventa erunt in Curva quarti gradus.

Etenim AB : BK :: AD : DR, & AB : BK :: RQ : QK; consequenter AD : DR :: RQ : QK.

$$\text{Igitur } 2a - x : x :: \sqrt{(2ax - x^2)} : \frac{x\sqrt{(2ax - x^2)}}{2a - x};$$

$$\text{ideoque } QK = \frac{x\sqrt{(2ax - x^2)}}{2a - x}. \text{ Quoniam ve-}$$

ro AB : BQ :: KQ : DP, erit $\sqrt{(4a^2 - 2ax)} :$

$$x :: \frac{x\sqrt{(2ax - x^2)}}{2a - x} : y; \text{ proindeque } y\sqrt{(4a^2 - 2ax)}$$

$$- 2ax) = \frac{x^2 \sqrt{(2ax - x^2)}}{2a - x}, y(2a - x)$$

$$\sqrt{(4a^2 - 2ax)} = x^2 \sqrt{(2ax - x^2)}, y^2(2a - x)^2 (4a^2 - 2ax) = x^4 (2ax - x^2),$$

$$2ay^2(2a - x)^3 = x^5(2a - x), 2ay^2(2a - x)^2 = x^5, \& x^5 - 2ay^2x^2 + 8a^2y^2x - 8a^3y^2 = 0.$$

Sit DP quarta proportionalis post AB, QK, & radium CR. Erit itaque $\sqrt{(4a^2 - 2ax)} : x\sqrt{(2ax - x^2)} :: a : y$. Quare $y\sqrt{(4a^2 - 2ax)}$

$$= \frac{ax\sqrt{(2ax - x^2)}}{2a - x}, y^2(2a - x)^2$$

$$(4a^2 - 2ax) = a^2x^2(2ax - x^2), 2ay^2x(2a - x)^3 = a^2x^3(2a - x), 2ay^2(2a - x)^2 = a^2x^3, ax^3 - 2y^2x^2 + 8ay^2x - 8a^2y^2 = 0.$$

Igitur Curva punctorum P est tertii gradus.

§. XI.

Ex quolibet puncto semiperipheriae B agatur tangens BK, quae occurrat diametro AR producto in K, accipiaturque DP tertia continue proportionalis ad CK, & BD; oportet invenire locum punctorum P. Fig. 29.

Quoniam angulus CBK est rectus, erit CD: CB::

$$CB : CK, \text{ idest } a - x : a :: a : \frac{a^2}{a - x}, \text{ ideo-}$$

$$\text{que } CK = \frac{a^2}{a - x}. \text{ Quare } \frac{a^2}{a - x} : \sqrt{(2ax - x^2)} :: \sqrt{(2ax - x^2)} : y; \& \text{ proinde } 2ax$$

$$-x^2 = \frac{a^2 y}{a - x}, x^3 - 3ax^2 + 2a^2 x - a^2 y$$

$= 0$, quae ad Curvam secundi generis pertinet.

Fiat DG media proportionalis inter CB, & DK.

$$\text{Quia } CK = \frac{a^2}{a - x}, \text{ erit } DK = \frac{a^2}{a - x} - a$$

$$+ x = \frac{2ax - x^2}{a - x}. \text{ Quapropter } y^2 = \frac{2a^2 x - ax^2}{a - x},$$

$ay^2 - y^2 x = 2a^2 x - ax^2$, atque $ax^2 - y^2 x - 2a^2 x + ay^2 = 0$; sunt igitur etiam puncta G in Curva secundi ordinis.

§. XII.

Fig. 30. Super radio AC describatur semicirculus AKC, ducaturque quaelibet recta AB, & demittantur perpendiculara BD, KH, fiatque AC:AB::HK:DP; locus punctorum P erit Curva secundi gradus.

Jungantur puncta B, R, & K, C rectis BR, KC. Quoniam anguli ABR, AKC recti existunt, erunt rectae BR, KC parallelae inter se; ideoque AB:AK::AR:AC. At vero AR est dupla AC; igitur etiam AB erit dupla AK. Est autem AB:AK::BD:KH; ergo erit quoque BD dupla rectae KH. Quamobrem $KH = \frac{1}{2}\sqrt{(2ax - x^2)}$. Jam vero AC:AB::HK:DP, nempe $a:\sqrt{(4a^2 - 2ax)}::\frac{1}{2}\sqrt{(2ax - x^2)}:y$; consequenter $ay = \frac{1}{2}\sqrt{(2ax - x^2)}\sqrt{(4a^2 - 2ax)}$, & $a^2 y^2 = \frac{1}{4}(2ax - x^2)(4a^2 - 2ax)$, $2a^2 y^2 = 4a^3 x - 4a^2 x^2 + ax^3$, $x^3 - 4ax^2 + 4a^2 x - 2ay^2 = 0$.

§. XIII.

Ducatur quaelibet recta KH tangens semicirculum Fig. 31. minorem CHR, fiatque DP tertia continue proportionalis post KC, & DB; puncta P eodem pacto determinata dabunt locum Curvae secundi generis.

Etenim cum sit ED: EH:: EH: EK, seu $\frac{1}{2}a$
 $— x: \frac{1}{2}a:: \frac{1}{2}a: \frac{a^2}{2a — 4x}$, erit $EK = \frac{a^2}{2a — 4x}$,
 & $CK = \frac{a^2}{2a — 4x} + \frac{1}{2}a = \frac{a^2 — ax}{a — 2x}$. Qua-
 re $\frac{a^2 — ax}{a — 2x}: \sqrt{(2ax — x^2)}:: \sqrt{(2ax — x^2)}:$

y ; ideoque $2ax — x^2 = \frac{a^2y — ayx}{a — 2x}$, & $(2ax — x^2)$

$(a — 2x) = a^2y — ayx$, $2a^2x — 5ax^2 + 2x^3$
 $= a^2y — ayx$, $2x^3 — 5ax^2 + ayx + 2a^2x —$
 $a^2y = 0$.

Accipiat DP aequalis perpendiculo RF. Quoniam rectae DH, RF sunt parallelae inter se, erit KD: DH:: KR: RF, seu, cum sit KD = KE

$— ED = \frac{a^2}{2a — 4x} — \frac{1}{2}a + x = \frac{2ax — 2x^2}{a — 2x}$,

$\frac{2ax — 2x^2}{a — 2x}: \sqrt{(ax — x^2)}:: \frac{ax}{a — 2x}:$

$\frac{ax\sqrt{(ax — x^2)}}{2ax — 2x^2}$; consequenter RF =

$\frac{ax\sqrt{(ax — x^2)}}{2ax — 2x^2} = \frac{a\sqrt{(ax — x^2)}}{2a — 2x}$. Quapropter

$y = \frac{a\sqrt{(ax — x^2)}}{2a — 2x}$, $y^2(2a — 2x)^2 = a^2(ax$

$— x^2)$, $4y^2(a — x)^2 = a^2x(a — x)$, $4y^2(a$

$\frac{a^2x}{a^2x} - x) = a^2x, 4ay^2 - 4xy^2 = a^2x, 4y^2x + a^2x - 4ay^2 = 0$. Igitur puncta P pertinent ad Curvam secundi generis.

§. XIV.

Fig. 32. Sint ABR, KEH duo semicirculi concentrici, & CB quilibet radius semicirculi majoris; demittatur ex puncto B perpendicularum BD, agaturque RE secans normalem BD in P: puncta P erunt in Curva tertii gradus.

Sit $RD = x$, $DP = y$, $AR = 2a$, & $KH = 2b$. Quia $CB:BD::CE:EQ$, idest $a:\sqrt{(2ax - x^2)}::b:\frac{b\sqrt{(2ax - x^2)}}{a}$, erit $EQ = \frac{b}{a} \times \sqrt{(2ax - x^2)}$. Similiter habetur $BD:DC::EQ:QC$, nempe $\sqrt{(2ax - x^2)}:a-x::\frac{b}{a}\sqrt{(2ax - x^2)}:\frac{ab - bx}{a} = QC$. Quare $RD:DP::RQ:QE$, scilicet $x:y::a - \frac{ab - bx}{a}:\frac{b}{a}\sqrt{(2ax - x^2)}$; ideoque $\frac{bx}{a}\sqrt{(2ax - x^2)} = ay - \frac{aby - byx}{a}$, $bx\sqrt{(2ax - x^2)} = a^2y - aby + byx$, & $2ab^2x^3 - b^2x^4 = a^4y^2 - 2ba^3y^2 + a^2b^2y^2 + 2ba^2y^2x - 2ab^2y^2x + b^2y^2x^2$, $b^2x^4 - 2ab^2x^3 + b^2y^2x^2 + 2ba^2y^2x - 2ab^2y^2x + a^2b^2y^2 - 2ba^3y^2 + a^4y^2 = 0$.

Fig. 26.

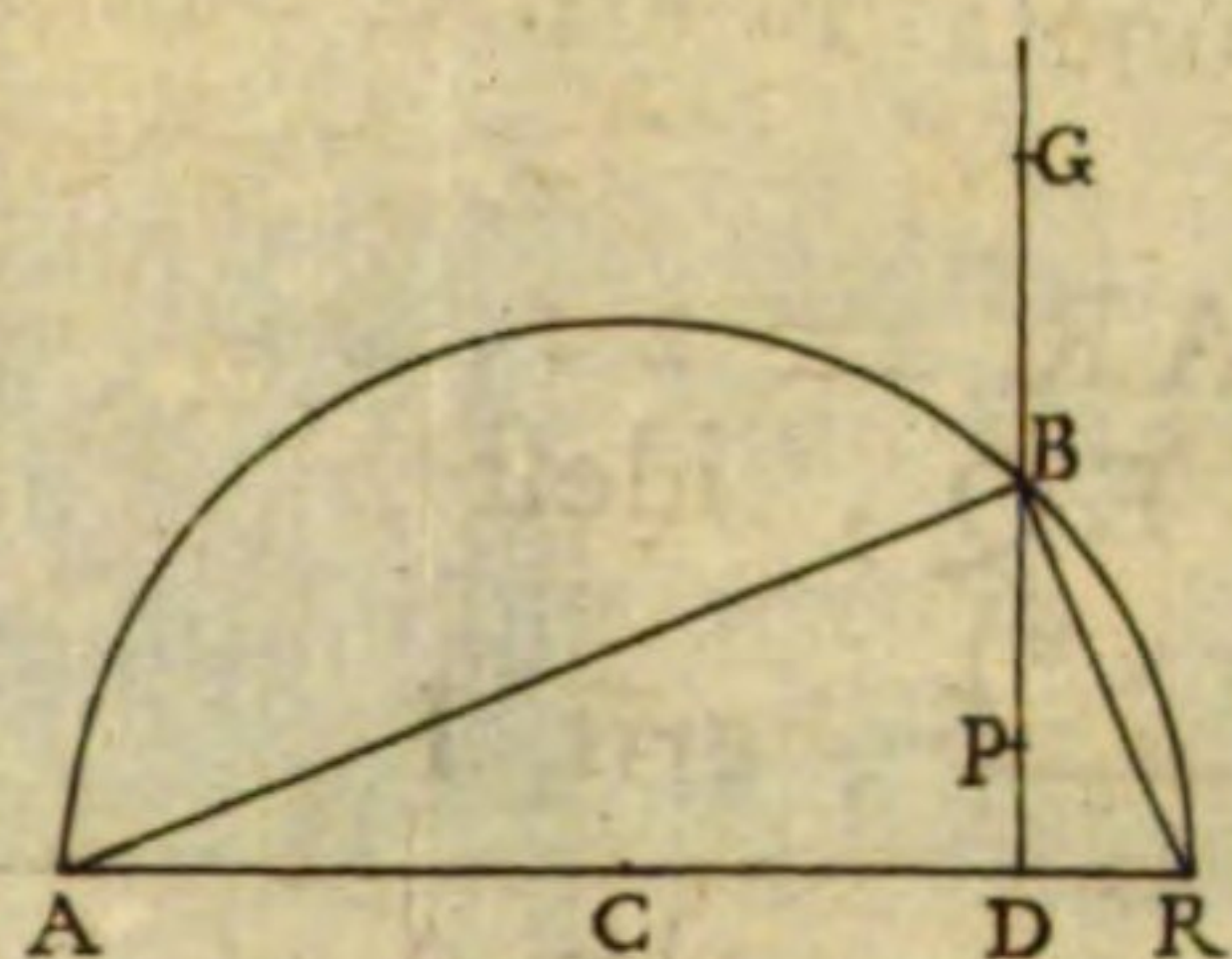


Fig. 28.

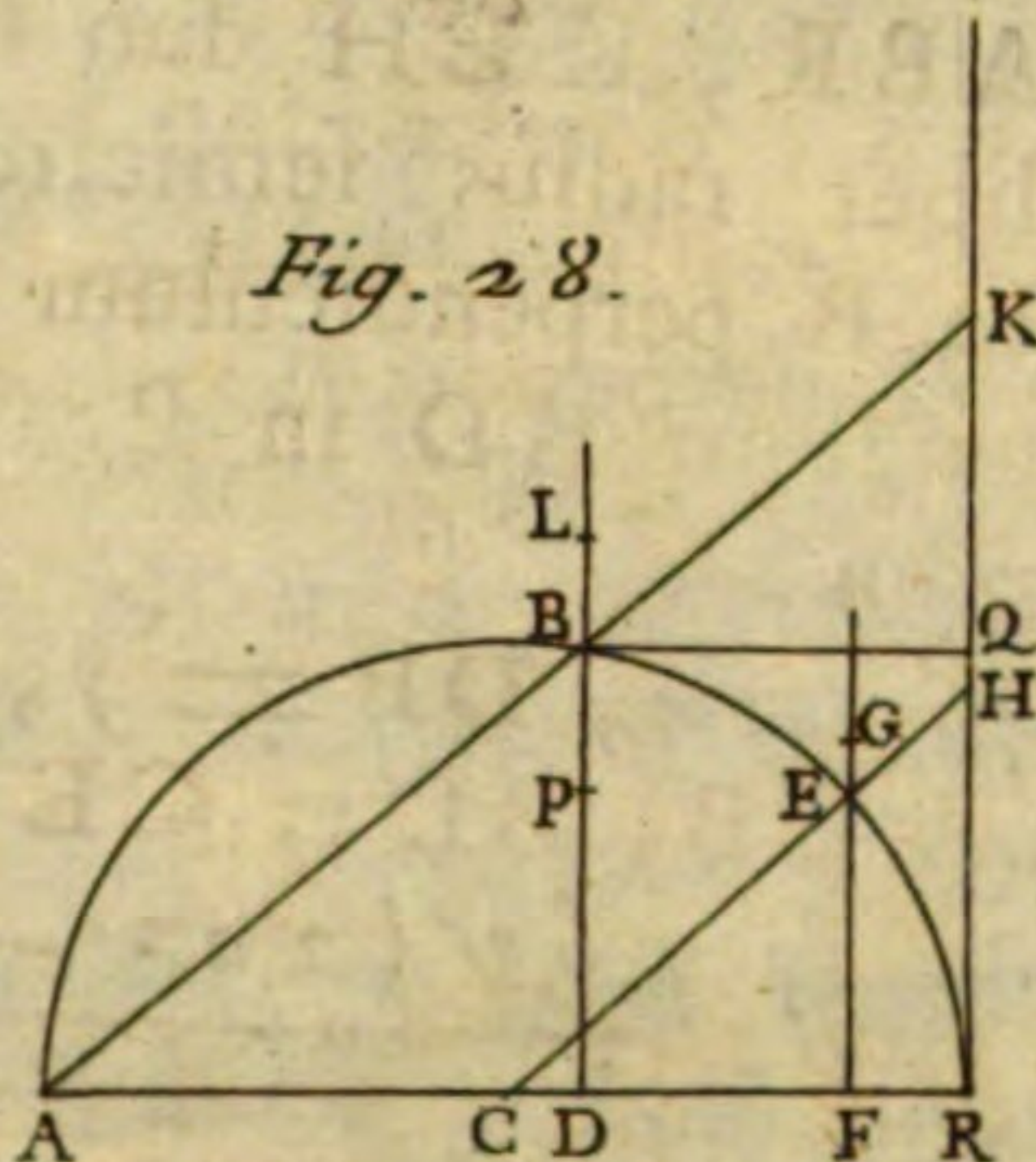


Fig. 30.

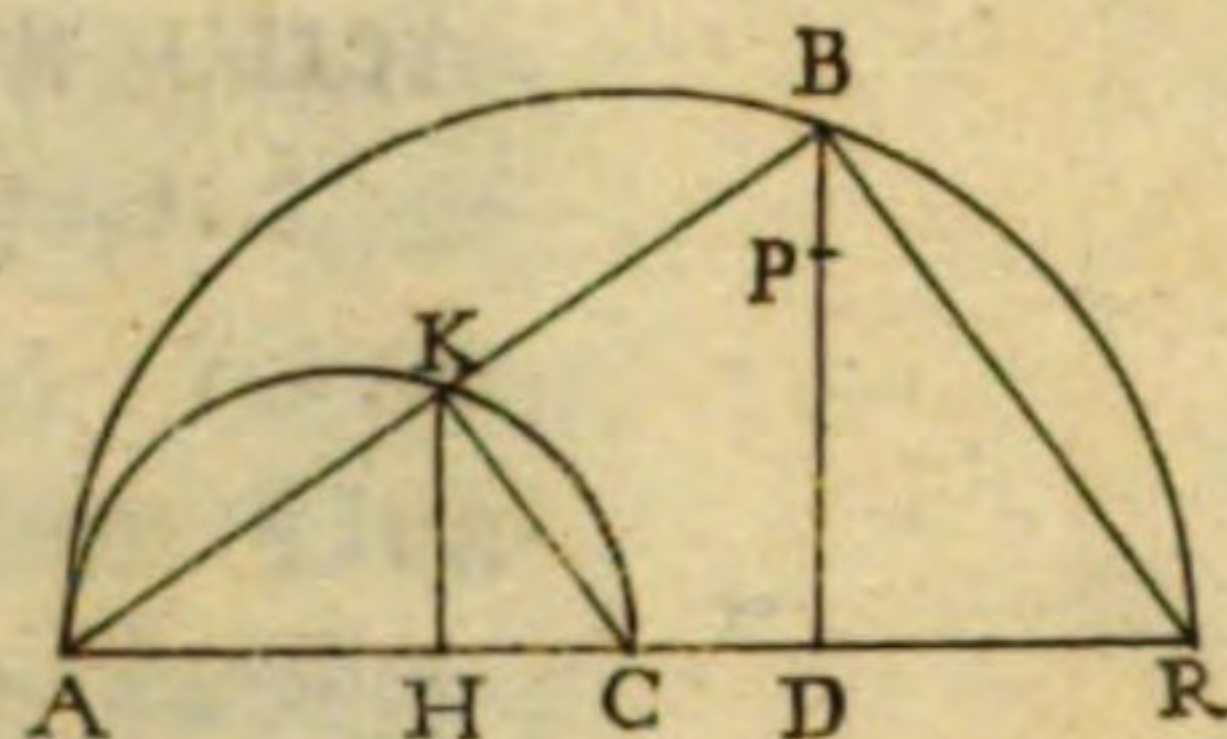


Fig. 27.

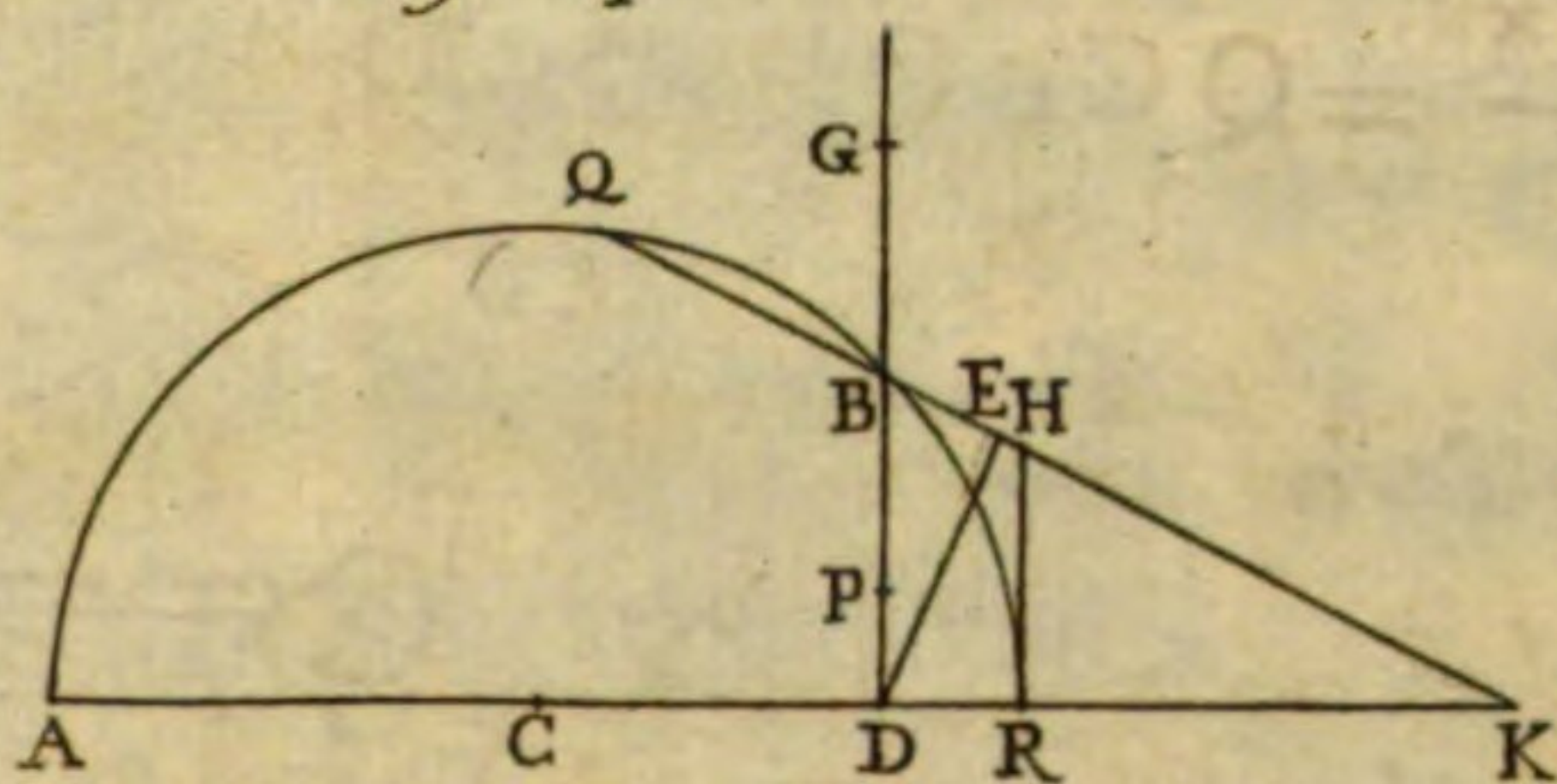


Fig. 29.

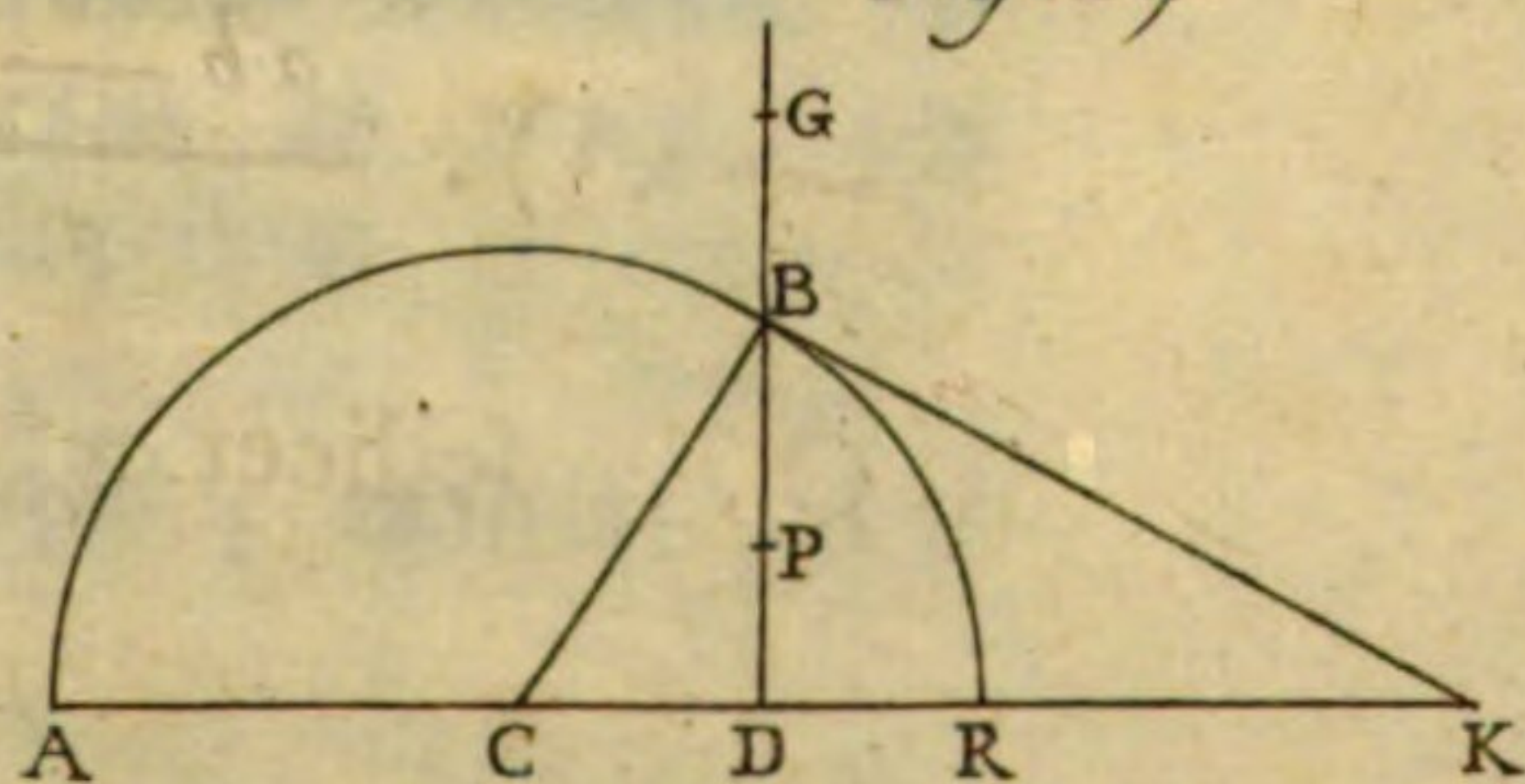


Fig. 31.

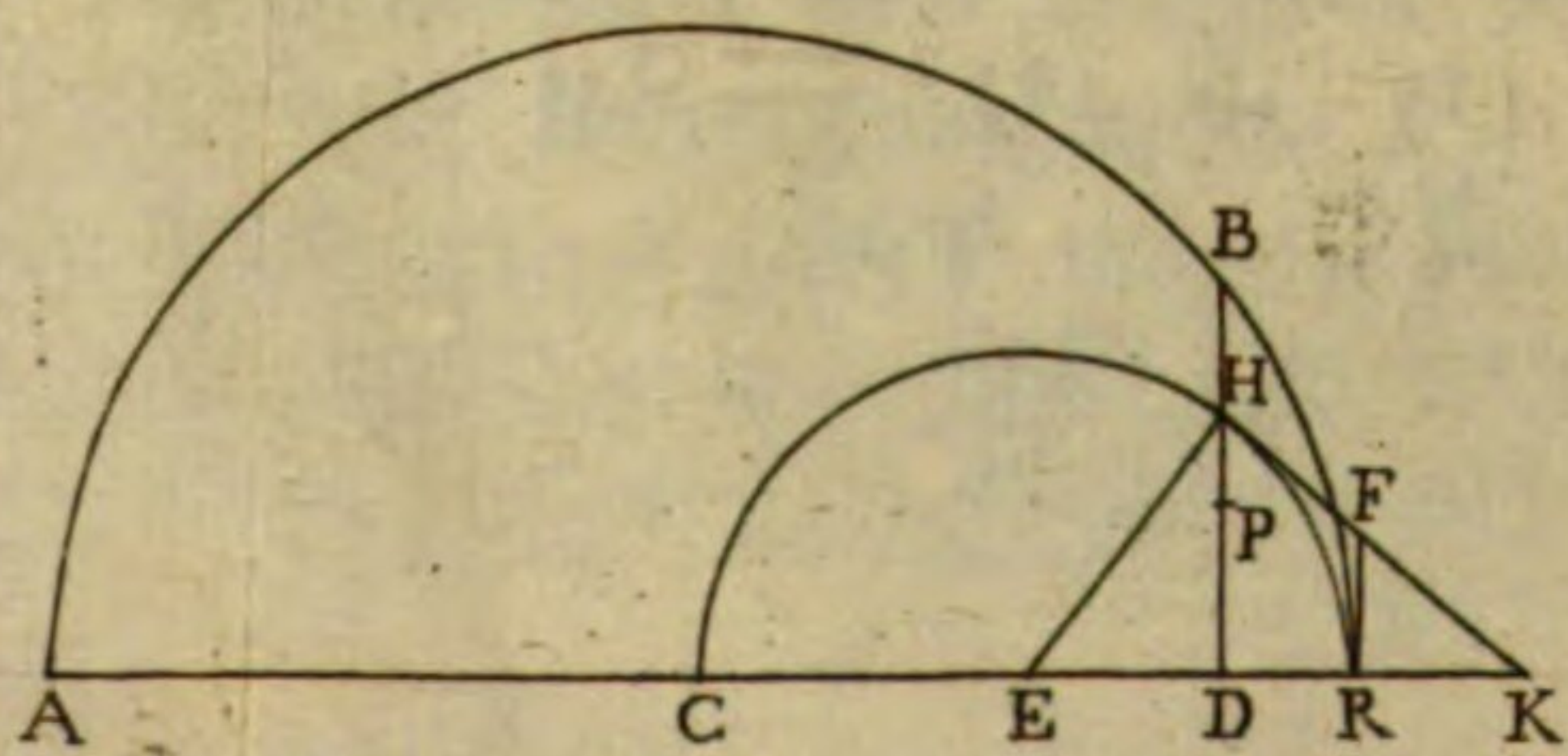
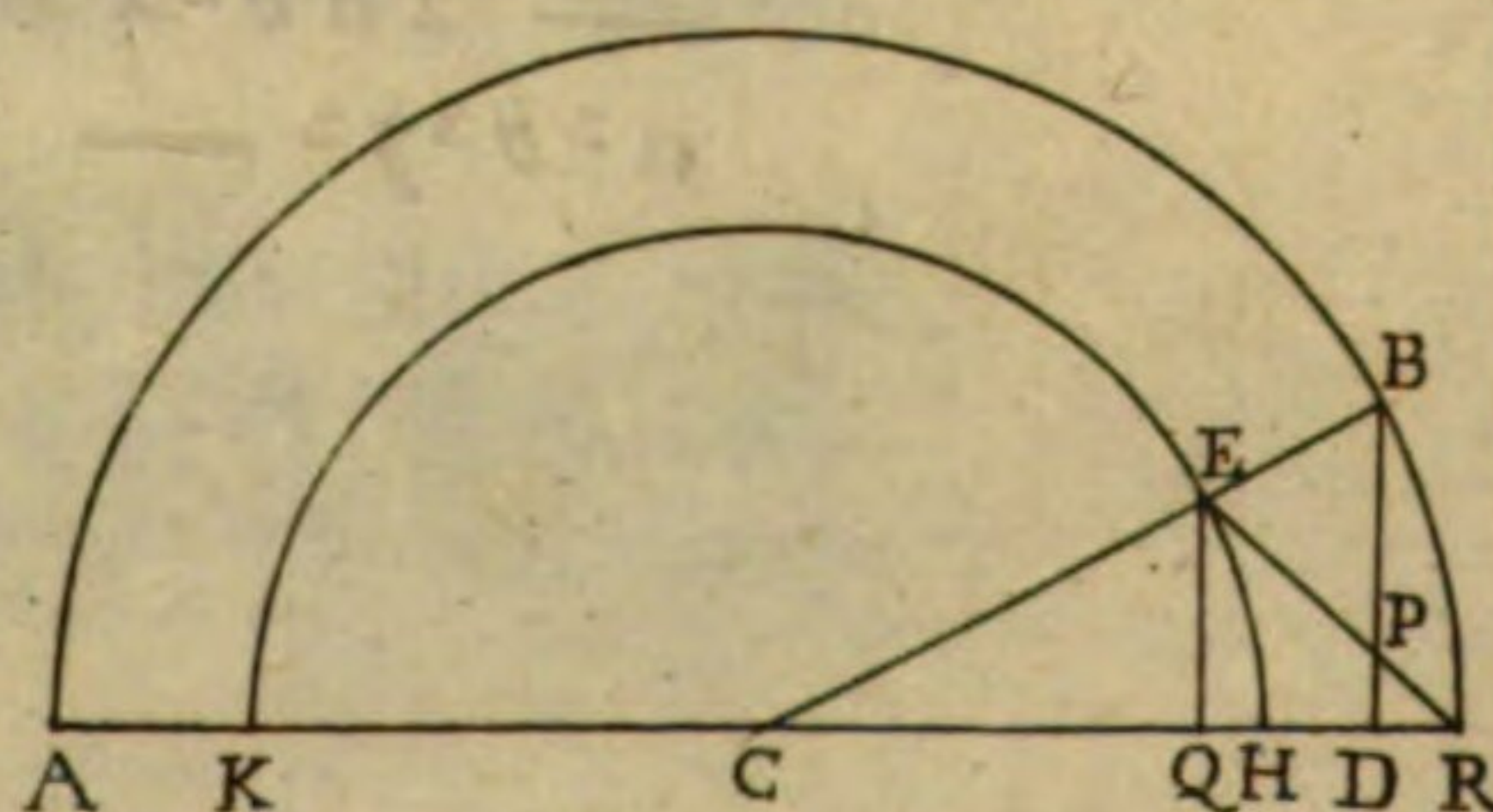
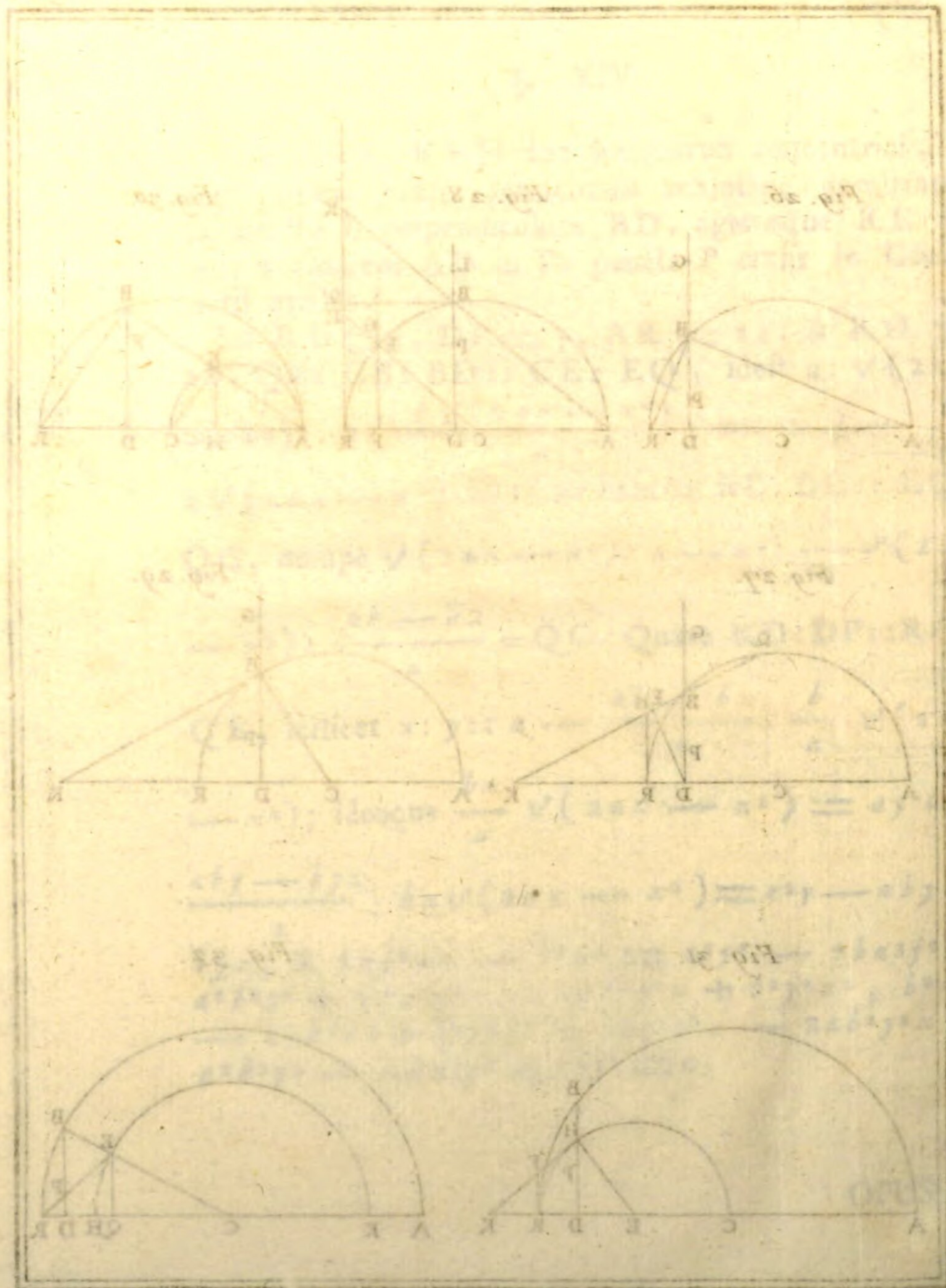


Fig. 32.





OPUSCULUM IV.

DE QUADAM OVALI SECUNDI GENERIS.



Uoniam in Opusculo superiori aequationes dumtaxat inveni, quae varios Curvarum gradus definiunt, non incongruum erit hoc in loco cujusdam Ovalis secundi generis nedum aequationem, verum etiam ejusdem incessum, atque subnormalem investigare, ut methodus, qua Curvae examinantur, eidem Ovali accommodata innotescat, quaque Curvas etiam superius inventas evolvere facile esset. Quamplurima nihilominus ad praedictam Ovalem pertinentia Analyseos Cultoribus invenienda relinquo, quandoquidem ob implicatas formulas, quae in iis perquirendis enascuntur, calculo plus aequo prolixo opus foret.

§. I.

Agatur in semicirculo QBR quaelibet chorda BR , *Fig. 33.* atque recta QK bifariam secetur in H , & jungatur RK ; tum ex puncto B demittatur perpendicularum BM , accipiaturque MP aequalis chordae RK : determinandus est locus punctorum P .

Sit $QM = x$, $MP = y$, & $QR = 2a$; erit $MR = 2a - x$, $QB = \sqrt{2ax}$, & $BR = \sqrt{4a^2 - 2ax}$. Quoniam BR secta est bifariam
H in

in H , erit $\frac{1}{2}BR = BH = \frac{1}{2}\sqrt{(4a^2 - 2ax)}$.

Quare $QH = BH + BQ$, & $QH = \sqrt{(BH^2 + BQ^2)} = \sqrt{(a^2 - \frac{1}{2}ax + 2ax)} = \sqrt{(a^2 + \frac{3}{2}ax)}$. Quamobrem $HQ : QB :: HR (= BH) : RK (= MP)$, nempe $\sqrt{(a^2 + \frac{3}{2}ax)} : \sqrt{2ax} :: \sqrt{(a^2 - \frac{1}{2}ax)} : y$; proindeque $y\sqrt{(a^2 + \frac{3}{2}ax)} = \sqrt{2ax} \times \sqrt{(a^2 - \frac{1}{2}ax)}$, & $a^2y^2 + \frac{3}{2}axy^2 = 2a^3x - a^2x^2$, $2ay^2 + 3xy^2 = 4a^2x - 2ax^2$, five $2ax^2 + 3y^2x - 4a^2x + 2ay^2 = 0$; consequenter puncta P dant locum Curvae secundi generis.

§. II.

Quoniam $3xy^2 + 2ay^2 = 4a^2x - 2ax^2$, erit $y : x :: 4a^2 - 2ax : 2ay + 3xy$, & $y : 2a - x :: 2ax : 3xy + 2ay$. Igitur semiordinata MP erit

ad abscissam QM , ut quadratum BR ad rectangulum ex applicata MP , & composita ex diametro QR , & tripla abscissa QM : pariter eadem semiordinata MP ad rectam MR eandem habebit rationem,

quam habet quadratum QB ad rectangulum ex applicata MP in praefatum aggregatum.

§. III.

Quia $y^2 = \frac{4a^2x - 2ax^2}{2a + 3x}$, erit $y = \pm$

$\sqrt{\left(\frac{4a^2x - 2ax^2}{2a + 3x}\right)}$, consequenter valor semiordinatae erit duplex, affirmativus nempe, & negativus,

vus, atque insuper in eodem abscissarum puncto semiordinata positiva erit aequalis privativae. Quapropter Curvae semiordinatae infra diametrum QR erunt aequales respondentibus applicatis supra eandem diametrum.

§. IV.

Sit $x = 0$; erit $2ay^2 = 0$, & $y^2 = 0$, seu $y = 0$, $y = 0$. Quare Curva secat axem in puncto Q, idest ad utramque partem in Q semiordinata nihilo aequatur. Fiat $x = 2a$; emerget $2ay^2 + 8ay^2 = 8a^3 - 8a^3$, nempe $y^2 = 0$; ideoque etiam in puncto R Curva trajicit axem. Et revera ponatur $y = 0$, erit $4a^2x - 2ax^2 = 0$, five $x^2 - 2ax = 0$; sed $x^2 - 2ax = 0$ oritur ex mutua multiplicatione binarum aequationum $x = 0$, & $x - 2a = 0$; ergo valor abscissae duplex erit, nempe $x = 0 = 2a$.

§. V.

Ponatur $x = -\frac{5}{2}a = -\frac{5}{2}a$; emergit $y = \sqrt{-5a^2} = \sqrt{-\frac{5}{19}a^2}$. Igitur si abscissa evadit negativa, seu major diametro QR, semiordinata fit imaginaria; consequenter Curva ultra extremitates diametri Q, & R non protrahitur: id quoque ex duobus superioribus §. §., sicuti ex ipsamet Curvae genesi satis clare deducitur.

§. VI.

Super radio CR describatur semicirculus CAR, at-
que ex puncto Q agatur tangens circulum QA, & producat in K, jungaturque KR; tum ex altero R

ducatur recta RB per contactum A transiens, & in perpendiculo BM accipiaturs recta PM aequalis KR : dico rectam PM esse maximam applicatam Curvae.

Rectam MP esse semiordinatam Curvae, quam superius reperimus, satis liquet, cum sit recta RB secata bifariam in A (§. XII. Opusc. III.). Quod vero sit maxima, sic probatur. Quoniam semiperipheria CAR bifariam dividit omnes rectas, quae ex puncto R ad dimidiam circumferentiam QBR duci possunt, omnia puncta A erunt in semiperipheria CAR . At vero angulus KQR , cujus latera sunt tangens QK , & diameter QR , major est quocunque alio HQR , cujus crura sunt eadem diameter QR , & quaelibet secans QH . Igitur etiam arcus KR excedet quemlibet alium HR ; consequenter chorda KR , seu applicata MP major erit qualibet alia HR .

§. VII.

Ex puncto contactus A agatur radius AD . Triangula QAD , QKR erunt similia. Quare $QD : DA :: QR : RK$. Sed DA est tertia pars rectae QD . Ergo etiam RK , seu maxima applicata MP erit aequalis trienti diametri QR . Praeterea cum sint rectae KR , AD parallelae inter se, erunt anguli alterni KRA , RAD aequales; sed sunt quoque aequales inter se anguli RAD , DRA , quia AD aequatur rectae DR ; igitur etiam angulus KRA erit aequalis angulo DRA . Quare, demisso perpendiculo AL , in trigonis AKR , ALR anguli KRA , LRA aequantur, item K , & ALR , atque latus AR est commune utrique triangulo; ideoque trigona AKR , ALR erunt aequalia inter se, consequenter etiam latus KR lateri LR aequabitur. At vero $RA : AB :: RL : LM$; proindeque recta RL erit aequalis

lis rectae LM, seu RM erit dupla rectae KR, idest
 aequalis duabus tertiis diametri QR. Igitur etiam
 abscissa QM aequabitur trienti diametri. Quapropter
 maxima semiordinata MP erit aequalis respondenti
 abscissae QM. Id vero methodo de maximis & mi-
 nimis amplius firmatur. Etenim differentiale aequatio-
 nis est $4aydy + 3y^2dx + 6xydy = 4a^2dx - 4axdx$,
 seu $4aydy + 6xydy = 4a^2dx - 4axdx - 3y^2dx$.
 Fiat $dy = 0$; erit $4a^2dx - 4axdx - 3y^2dx = 0$,
 $4a^2 - 4ax - 3y^2 = 0$, &
 $y^2 = \frac{4a^2 - 4ax}{3}$. At vero $y^2 = \frac{4a^2x - 2ax^2}{2a + 3x}$.

Quare $\frac{4a^2 - 4ax}{3} = \frac{4a^2x - 2ax^2}{2a + 3x}$, $8a^3 - 8a^2x + 12a^2x - 12ax^2 = 12a^2x - 6ax^2$,
 $8a^3 - 8a^2x - 6ax^2 = 0$, $x^2 + \frac{4}{3}ax = \frac{4}{3}a^2$, $x^2 + \frac{4}{3}ax + \frac{16}{36}a^2 = \frac{4}{3}a^2 + \frac{16}{36}a^2 = \frac{16}{9}a^2$,
 $x + \frac{2}{3}a = \sqrt{\frac{16}{9}a^2} = \frac{4}{3}a$, & $x = \frac{4}{3}a - \frac{2}{3}a = \frac{2}{3}a$. Substituatur modo
 in aequatione valor inventus abscissae; emerget $2ay^2 + 2ay^2 = \frac{8}{3}a^3 - \frac{8}{9}a^3 = \frac{16}{9}a^3$,
 $4ay^2 = \frac{16}{9}a^3$, $y^2 = \frac{4}{9}a^2$, & $y = \frac{2}{3}a = \frac{1}{3} \times 2a$.

§. VIII.

Quia recta quaelibet QH binis in punctis V, & S
 secat semiperipheriam CAR, secabit quoque bifariam
 binas chordas RF, RE; consequenter applicata NG
 erit aequalis semiordinatae TO, & intersectiones ar-
 cus CVA dabunt puncta segmenti Curvae QGP,
 alterius vero RSA puncta reliqui segmenti POR.

§. IX.

Jungatur QP, & demittatur perpendicularum VI.
 Et quoniam applicata NG eandem habet rationem
 H 3 ad

ad abscissam QN , quam habet quadratum RF ad
 rectangulum ex femiordinata NG , & composita ex
 diametro QR , & tripla abscissa QN (§. II.); erit
 quoque $NG : QN :: \frac{1}{4} RF : \frac{1}{4} NG \times (QR + 3QN)$.
 At vero QH dividit bifariam rectam RF ; igitur
 $\frac{1}{4} RF$ aequabitur quadrato RV , idest rectangulo ex
 CR in RI . Quare erit $NG : QN :: CR \times RI :$
 $\frac{1}{4} NG \times (QR + 3QN)$, seu $NG : QN :: 2RD$
 $\times RI : NG \times (RD + \frac{3}{4}QN)$. Atqui recta RI , us-
 quedum definitur a perpendicularo VI demisso ex quo-
 cunque puncto V intersectionis arcus AC , & recta-
 rum RF segmentum Curvae QGP determinantium,
 major est recta RL , idest RK (§. VII.), quae ma-
 ximae femiordinatae MP aequatur; igitur RI multo
 major erit applicata NG . Sed etiam RD major est
 $\frac{3}{4}QN$; etenim cum sit quaelibet abscissa QN ad se-
 gmentum Curvae QGP pertinens minor QM , sive
 $\frac{4}{3}RD$ (§. VII.), erit quoque $\frac{3}{4}QN$ minor $\frac{3}{4} \times \frac{4}{3}RD$,
 seu RD . Quamobrem antecedens $2RD \times RI$ major
 erit consequente $NG \times (RD + \frac{3}{4}QN)$, proindeque
 etiam femiordinata NG excedet abscissam QN . At
 vero $PM : MQ :: ZN : NQ$; ideoque cum sint PM ,
 & MQ aequales inter se, etiam ZN aequabitur ab-
 scissae NQ . Quare applicata NG major erit recta
 NZ ; consequenter segmentum Curvae QGP axi
 QM concavitatem obvertit.

§. X.

Agantur modo binae rectae RP , & QE . Quia
 femiordinata TO est ad rectam TR , ut quadratum
 QE ad rectangulum ex applicata TO , & composita
 ex

ex diametro QR , & tripla abscissa QT (§. II.);
 erit etiam $TR : TO :: TO \times (QR + 3QT) : QR$
 $\times QT$, & $\frac{1}{2}TR : TO :: \frac{1}{2}TO \times (QR + 3QT) :$
 $QR \times QT$, denique $\frac{1}{2}TR : TO :: \frac{1}{2}TO \times (QM +$
 $QT) : QM \times QT$ (§. VII.), seu $\frac{1}{2}TR : TO ::$
 $TO \times (\frac{1}{2}QM + \frac{1}{2}QT) : QM \times (\frac{1}{2}QT + \frac{1}{2}QT)$.
 At vero QM aequatur maximae semiordinatae MP ,
 ideoque excedit quamlibet applicatam TO ; & quae-
 libet abscissa QT ad segmentum Curvae POR spe-
 ctans excedit QM , consequenter etiam $\frac{1}{2}QT$ major
 est $\frac{1}{2}QM$. Quare antecedens $TO \times (\frac{1}{2}QM + \frac{1}{2}QT)$
 minor erit consequente $QM \times QT$, proindeque etiam
 $\frac{1}{2}TR$ minor erit applicata TO . Sed $RM : MP ::$
 $RT : TX$, & MP est dimidium rectae RM (§. VII.);
 igitur TX erit quoque aequalis $\frac{1}{2}RT$. Quamobrem
 recta TX minor erit semiordinata TO ; & conse-
 quenter etiam alterum Curvae segmentum POR ad
 axem MR concavitatem flectit.

§. XI.

Quamobrem tum ex haecenus demonstratis, tum
 ex ipsamet Curvae genesi tuto inferri potest, jugiter
 axi QR Curvam nostram QPR concavitatem obver-
 tere; ideoque considerari potest instar Ovalis $QPRYQ$
 in se redeuntis, in qua tres rectae MP , MQ , &
 MY aequales sunt inter se, & axis minor PY ad
 majorem QR eandem habet relationem, quam 2
 ad 3.

§. XII.

Crescentibus igitur abscissis a Q usque ad M , idest
 donec abscissa QM semiordinatae MP fiat aequalis,
 crescunt quoque applicatae, quarum quaelibet respon-
 den-

dentem abscissam excedit (§. IX.); ultra vero punctum M crescentibus abscissis semiordinatae minuuntur, iisdemque minores evadunt.

§. XIII.

Fig. 35. Sit recta MP quaelibet applicata Ovalis; producat diameter RQ in B tali pacto, ut sit QB aequalis $\frac{3}{2}$ QM, & describatur semicirculus BFR; tum dividatur bifariam abscissa QM in N, ex quo erigatur normalis NF, & applicetur in minori semicirculo QKR recta QK aequalis chordae BF, jungaturque KR; deinde ex puncto R elevetur recta RD perpendicularis diametro RQ, & aequalis rectae RK, atque protrahatur diameter RB donec fiat BA aequalis segmento BQ, & jungatur AD; tandem accipiat MO aequalis radio QC, atque ex O agatur OE parallela rectae AD, erigaturque ex puncto E recta EG perpendicularis rectae OE: dico rectam GP esse Curvae normalem in puncto P.

Sit DL, quae occurrit diametro QR productae in L, perpendicularis rectae AD. Quoniam rectae OE, EG sunt parallelae rectis AD, DL, trigona OEG, ADL erunt similia; consequenter OM:MG::AR:

RL. At vero $AR:RL::AR^2:RD^2 (=RK^2)$, quia tres rectae AR, RD, & RL sunt in continua

proportione geometrica. Quare OM:MG:: AR^2 :

RK^2 . sed $RK^2 = QR^2 - QK^2 = QR^2 - BF^2$

$= QR^2 - BN \times BR$. Igitur erit OM:MG::

$AR^2:QR^2 - BN \times BR$. Quamobrem cum sit BQ $= \frac{3}{2}QM = \frac{3}{2}x$, QN $= \frac{1}{2}QM = \frac{1}{2}x$, & AQ

=

$$\begin{aligned}
&= 2 B Q = 3 x; \text{ erit } a : M G :: (2 a + 3 x)^2 : 4 a^2 \\
&\quad - 2 x \left(\frac{3}{2} x + 2 a \right), \text{ seu } (2 a + 3 x)^2 : 4 a^2 - \\
&\quad 3 x^2 - 4 a x :: a : M G; \text{ proindeque } M G = \\
&\quad \frac{4 a^3 - 3 a x^2 - 4 a^2 x}{(2 a + 3 x)^2}. \text{ Verum } 4 a y d y + 6 x y d y \\
&= 4 a^2 d x - 4 a x d x - 3 y^2 d x; \text{ ideoque } \frac{y d y}{d x} \\
&= \frac{4 a^2 - 4 a x - 3 y^2}{4 a + 6 x} = \frac{4 a^3 - 3 a x^2 - 4 a^2 x}{(2 a + 3 x)^2}.
\end{aligned}$$

Ergo $M G$ aequatur $\frac{y d y}{d x}$, nempe subnormali, consequenter $G P$ erit perpendicularis Curvae in puncto P ; ideoque si agatur per punctum P normalis eidem rectae $G P$, ipsamet Ovalem continget in puncto P .

§. XIV.

Si recta $B F$ emerget aequalis diametro $Q R$, tunc subnormalis $M G$ nihilo aequabitur, ideoque tangens erit parallela axi $Q R$: id autem accidet, cum abscissa fiet aequalis trienti diametri $Q R$, quo in casu maximae semiordinatae respondet.

§. XV.

Verum si recta $B F$ fuerit major diametro $Q R$, *Fig. 36.* quod eveniet, quoties normalis ducenda est ad alterum Curvae segmentum ultra maximam applicatam, tunc accipienda est recta $R S$ aequalis eidem rectae $B F$, & in semicirculo $S K R$ applicari oportet rectam $S K$ aequalem diametro $Q R$: caetera vero peragenda sunt ut in §. XIII. Proveniet enim $M G =$
 $4 a^2 x$

$\frac{4a^2x + 3ax^2 - 4a^3}{(2a + 3x)^2}$. At vero recta subnormalis

aequatur $\frac{4a^3 - 4a^2x - 3ax^2}{(2a + 3x)^2}$, nempe eidem re-

ctae MG, sed signo privativo affectae. Quare ex parte opposita sumenda est recta MH aequalis rectae MG. Si igitur ducatur HP, haec erit normalis Curvae in puncto P.

F I N I S.

Fig. 34.

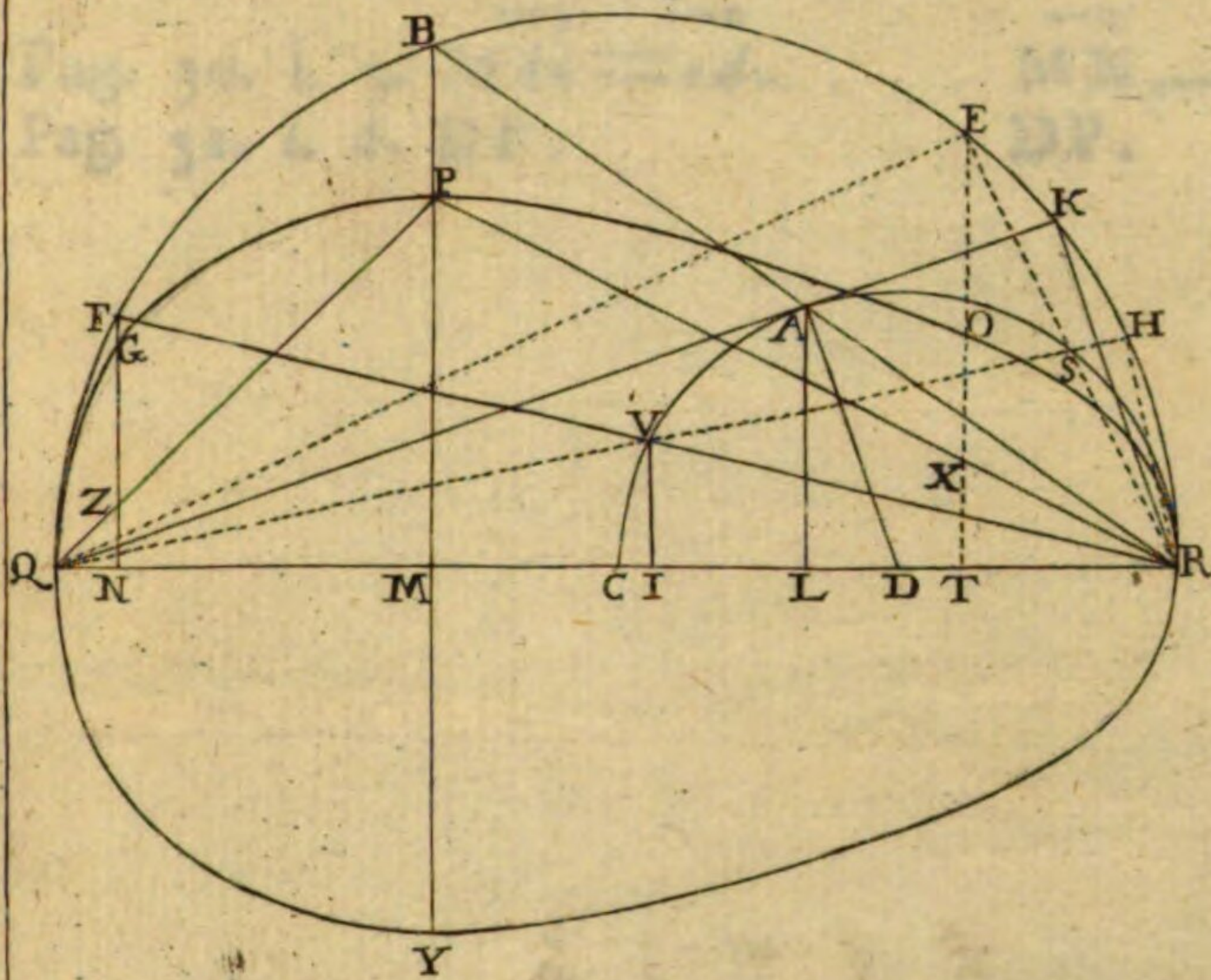


Fig. 33.

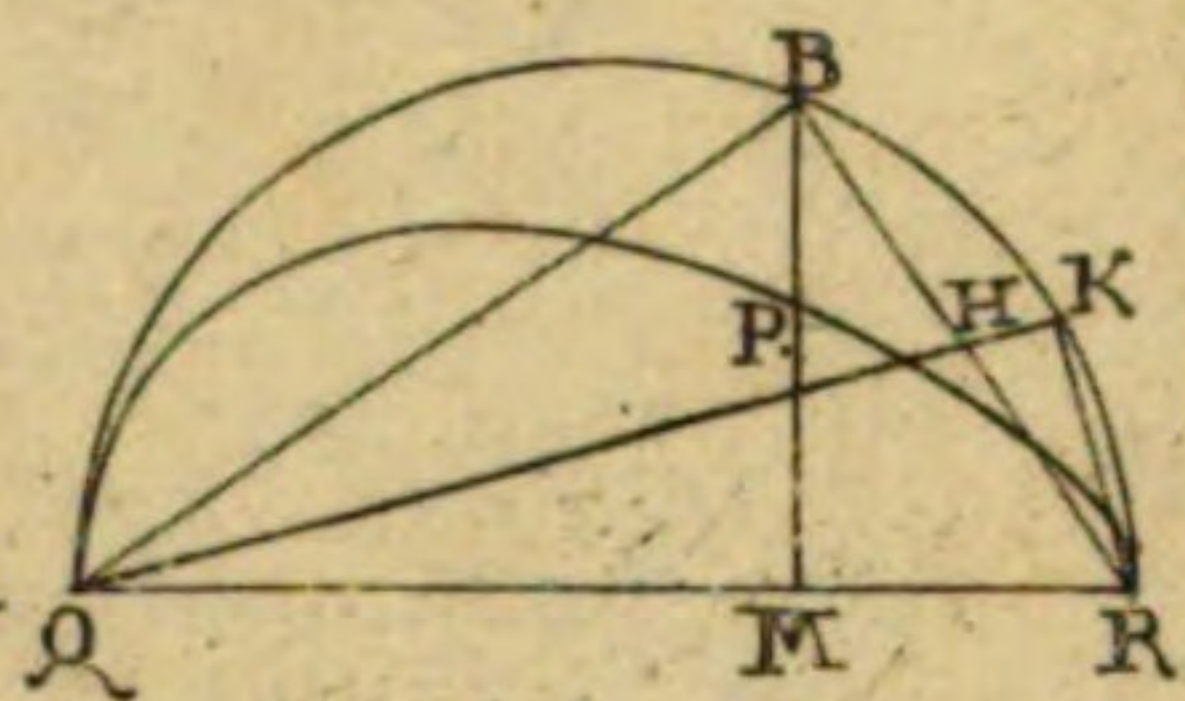


Fig. 36.

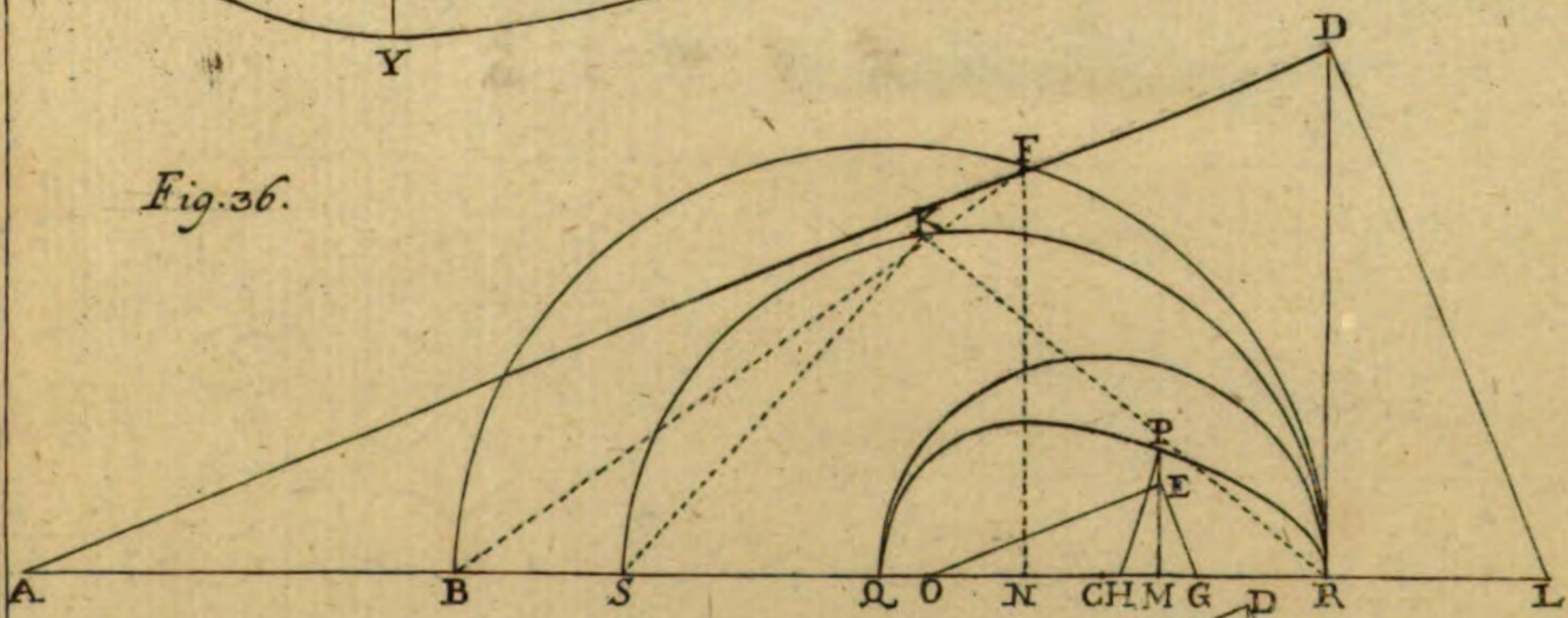
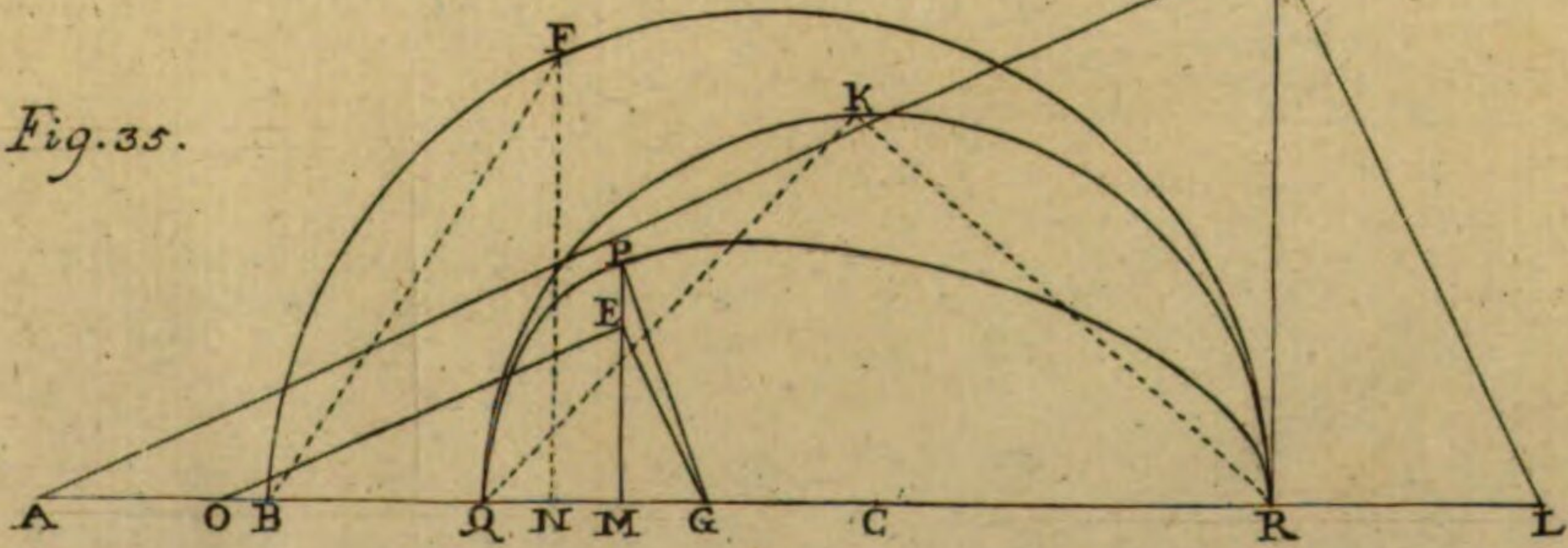
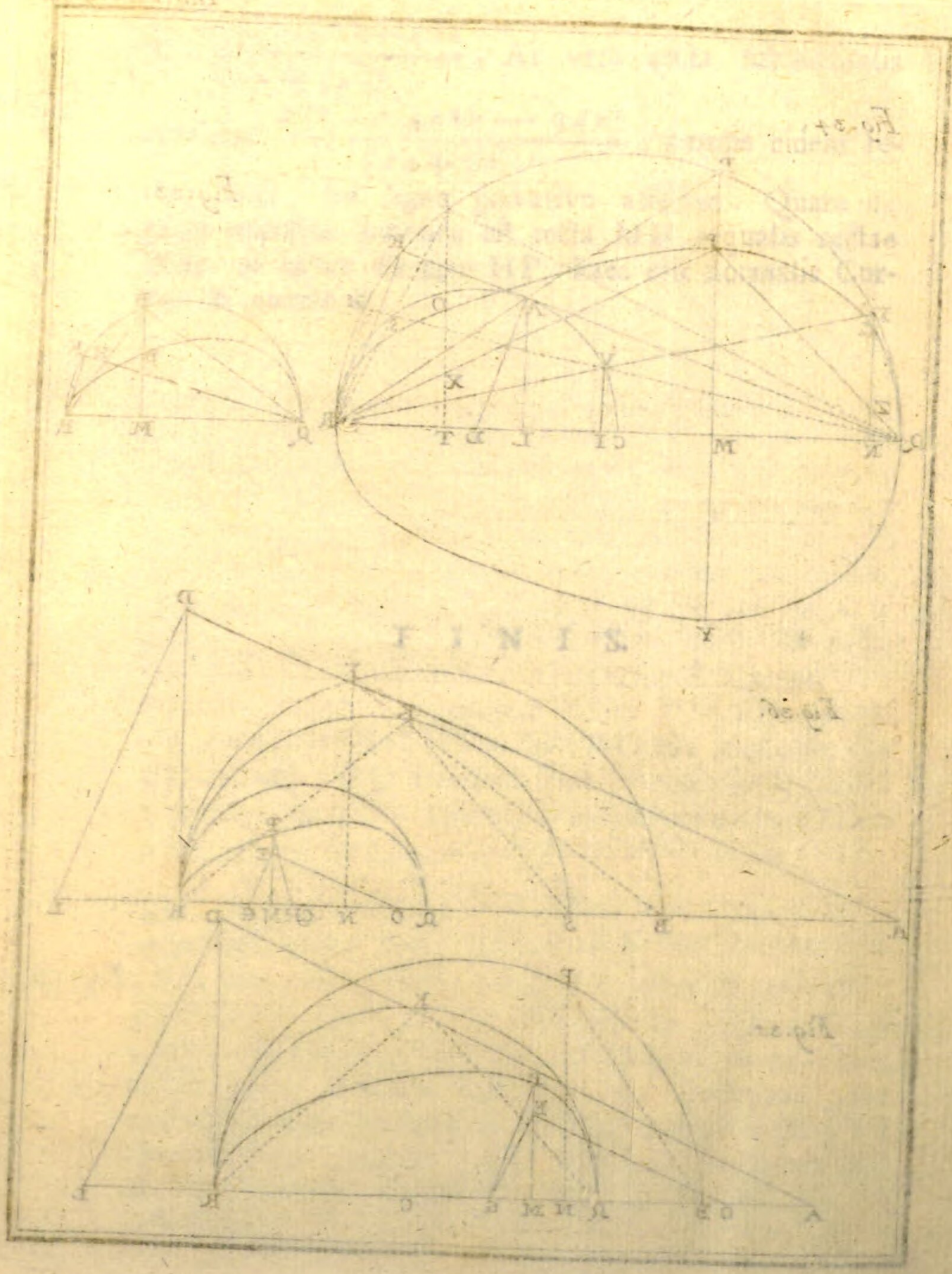


Fig. 35.



Tab. VII. 3. 66



ERRATA

CORRIGE

Pag. 16. l. 5. fcalena,

Pag. 17. l. 9. BN::DP,

Pag. 30. l. 5. $\overset{-2}{M}\overset{-2}{N} = cd.$

Pag. 51. l. 6. DP:

fcalena,

BN:DP,

$\overset{-2}{M}\overset{-2}{N} \text{ --- } cd.$

DP.

ERRATA

CORRIGE

Page 21. l. 6. DP:
Page 30. l. 2. MN = ca.
Page 17. l. 9. BN: DP,
Page 16. l. 2. scilena,

DP.
MN — ca.
BN: DP,
scilena,

